Date Due

~~AUG 3 1969~~			
~~N... 976~~			
~~FEB 2 1 1979~~			

23233 PRINTED IN U.S.A.

LONGMANS' CIVIL ENGINEERING SERIES

CALCULATIONS

IN

HYDRAULIC ENGINEERING

LONGMANS'

CIVIL ENGINEERING SERIES

CIVIL ENGINEERING AS APPLIED IN CONSTRUCTION.

By LEVESON FRANCIS VERNON-HARCOURT, M.A., M.Inst.C.E., Author of "Rivers and Canals," etc. With 368 Illustrations. 8vo, 14*s. net.*

NOTES ON DOCKS AND DOCK CONSTRUCTION.

By C. COLSON, M.Inst.C.E., Deputy Civil Engineer-in-Chief, Admiralty. With 365 Illustrations. Medium 8vo, 21*s. net.*

CALCULATIONS IN HYDRAULIC ENGINEERING: A Practical Text-Book for the use of Students, Draughtsmen, and Engineers.

By T. CLAXTON FIDLER, M.Inst.C.E., Professor of Engineering, University College, Dundee.

Part I. Fluid Pressure and the Calculation of its Effects in Engineering Structures. With numerous Illustrations and Examples. Medium 8vo, 6*s.* 6*d. net.*

Part II. Calculations in Hydro-kinetics.

RAILWAY CONSTRUCTION.

By W. H. MILLS, M.I.C.E., Engineer-in-Chief of the Great Northern Railway of Ireland. With 516 Illustrations and Diagrams. Medium 8vo, 18*s. net.*

PRINCIPLES AND PRACTICE OF HARBOUR CONSTRUCTION.

By WILLIAM SHIELD, F.R.S.E., M.Inst.C.E., and Executive Engineer, National Harbour of Refuge, Peterhead, N.B. With 97 Illustrations. Medium 8vo, 15*s. net.*

TIDAL RIVERS: their (1) Hydraulics, (2) Improvement, (3) Navigation.

By W. H. WHEELER, M.Inst.C.E. With 75 Illustrations. Medium 8vo, 16*s. net.*

LONGMANS, GREEN, AND CO.
LONDON, NEW YORK, AND BOMBAY

LONGMANS' CIVIL ENGINEERING SERIES

CALCULATIONS

IN

HYDRAULIC ENGINEERING

A PRACTICAL TEXT-BOOK FOR THE USE OF STUDENTS, DRAUGHTSMEN, AND ENGINEERS

WITH NUMEROUS ILLUSTRATIONS AND EXAMPLES

BY

T. CLAXTON FIDLER, M.INST.C.E.
PROFESSOR OF ENGINEERING, UNIVERSITY COLLEGE, DUNDEE,
UNIVERSITY OF ST. ANDREWS

PART II.

CALCULATIONS IN HYDRO-KINETICS

LONGMANS, GREEN, AND CO.
39 PATERNOSTER ROW, LONDON
NEW YORK AND BOMBAY
1902

PREFACE

THE calculations which form the subject-matter of this volume are those relating to the motion of water and to the design of works intended for its conveyance. The problems that most frequently arise in the design of such works are treated from a practical point of view; and the chief object of the writer throughout has been to meet the actual requirements of the hydraulic engineer in his daily practice.

In great measure these problems will have reference to the uniform flow of water in pipes and channels, and in all such questions it must be admitted that nothing like a mathematical accuracy is by any means attainable; but it will be the desire of every engineer to employ a method that shall approximate as closely as possible to the ascertainable truth. On the other hand, such calculations will often have to be repeated many scores or hundreds of times for the purposes of one work of construction; and the exigencies of an engineer's practice will almost forbid the employment of any method that involves a great complexity of calculation. The methods which have been followed in this volume will generally require the use of a table of logarithms; but if they do not quite possess the delightful simplicity of the old and well-known formula, they will certainly avoid its more serious errors, while they are capable of being applied with great facility to the questions that most frequently arise in practice. To lighten the labour of such computations, tables of discharge have been worked out for pipes and for brick culverts of circular section.

At the same time, the evidences of experiment, including recent observations, have been analyzed with the object of determining the varying values of the coefficient in the old formula, so that the familiar calculations can be worked out with a tolerable degree of accuracy, on the old lines, wherever that course may be preferred.

Another and quite a different class of problems will relate to

the motion of water as depending upon its calculable acceleration under the action of gravity; and here, of course, the question admits of a more positive solution upon well-known principles.

It would be impossible to deal with a subject of this kind without reference to information which has been contributed by writers and observers of all civilized countries, and has been embodied in a literature which extends over more than two centuries of time. To trace the accumulated evidences of experiment, and the historical development of theory, frequent reference has been made to the works of Hagen and Weisbach, of D'Arcy, Bazin, Ganguillet, and Kutter, of Beardmore, Hamilton Smith, Fanning, Osborne Reynolds, Downing, and Greenhill, as well as to the recorded observations of many engiueers and hydraulicians, whose names are quoted throughout the following pages.

T. C. F.

UNIVERSITY COLLEGF, DUNDEE,
UNIVERSITY OF ST. ANDREWS,
February, 1902.

CONTENTS

PART II.

CALCULATIONS IN HYDRO-KINETICS.

CHAPTER I.

THE FLOW OF WATER.

CHAPTER II.

EXPERIMENTS ON THE FLOW OF WATER IN PIPES.

CHAPTER III.

EXPERIMENTS ON THE FLOW IN OPEN CHANNELS.

CHAPTER IV.

DETERMINATION OF CONSTANTS IN A GENERAL FORMULA.

CHAPTER V.

CALCULATIONS RELATING TO UNIFORM FLOW.

CHAPTER VI.

INLETS AND OUTLETS.

PART II.

CALCULATIONS IN HYDRO-KINETICS.

CHAPTER I.

THE FLOW OF WATER.

Art. 1.—In dealing with any problem that relates to the static pressure of water, the engineer can generally proceed by purely rational methods, whose results are free from any doubt or ambiguity. His calculations are based upon a few natural principles which are universally true and are sufficiently simple to admit of an easy application; and when he has once ascertained the weight of a cubic foot of water, he needs no other experimental observation of any kind.

But as soon as we enter upon the subject of water in motion, we immediately find that engineering calculations have to be carried out by quite different methods, in which empirical rules and experimental coefficients take the place of a rational theory; and the transition illustrates very forcibly the contrast between the rational and the experimental method of investigation. It is probably true that every particle of water in a stream moves under the guidance of fixed laws; and, if we could trace out the influence of all the surrounding conditions, these laws should perhaps enable us to predict the changing movements of every ripple and eddy, and the broader motion of the whole stream. But the movements, the laws, and the surrounding conditions are all very complex; and it may even be doubted whether they are capable of being expressed in any definite mathematical form. A great deal of labour has certainly been devoted to the mathematical theory of hydro-dynamics; and yet, for the practical

purposes of engineering, its results are almost worthless, and sometimes they are at variance with the plainest facts.

Starting with its own definitions as to the mechanical properties of "a fluid," the theory shows incontestibly that certain consequences must follow:—a sphere, moved through the fluid at any given velocity, would experience no resistance to its motion, however great the velocity might be; ships would travel at an infinite speed, propelled by a power infinitesimally small; and the rainfall would, no doubt, rush from the mountains to the sea in the same mad haste, leaving the land in a permanent condition of drought.

It is, of course, impossible for the engineer to rely upon a theory which presents to him such conclusions as these. They may be true of some imaginary fluid which exists only in the imaginary world of hydro-dynamics; but they have no application to the world we live in.

Hence it has come to pass that hydraulic engineers have found but little help from rational theory,—employing it only in a few of their calculations; and for the rest they have been compelled to direct their practice by the light of observed facts.

This alternative method of procedure is, however, not only more troublesome, but also more hazardous, because of the immense number of facts that must be collected before any safe conclusions can be drawn from them.

During the last two centuries many hundreds, or perhaps thousands, of experiments have been made by observers in various parts of the world, with the object of measuring the flow of water in pipes, channels, and natural streams, under all sorts of varying conditions; and yet they are all too few for the purpose.

At every period, however, engineers have found themselves under the necessity of carrying forward their works of construction as best they could with the information that was then available; and, for the practical purposes of their work, they have tried to express the results of experiment, so far as they were known, by means of some "empirical" formula.

Naturally enough, the formula has taken many shapes. At first the experiments were few, and their apparent results were conveyed in a simple mathematical expression, which was based upon insufficient evidence. But as the recorded observations increased in number and in completeness, they revealed the effect of certain conditions whose influence had hitherto been unsuspected.

For the velocity of the current which flows in any conduit will be governed by a number of conditions, which vary in each individual case. It will certainly depend upon the inclination, of the conduit, upon the form and dimensions of its cross-section, and upon the roughness or smoothness of its walls. And the true business of the "empirical," or experimental, method is to discover in *what way* and to *what extent* the question is affected by each one of these matters taken separately—how it is affected when we vary the inclination of the conduit, when we alter the form or the dimensions of its cross-section, when we make any change in the roughness of the walls.

These questions can only be settled by experimental tests carried out in successive series, or groups of series, and so arranged as to bring under examination the effect of varying *one* of these conditions, *without* varying any of the others at the same time. At a later stage, we shall have to consider the evidence that can be gleaned from such experiments; but, in the present chapter, it will only be possible to obtain a general view of the field that we have to travel over.

From what has already been said, it will be seen, in a general way, how the land lies, and what will be the character of the inquiry. We shall often be engaged in piecing together the evidences of experimental gaugings, and we shall find only a very limited scope for the application of any exact scientific principles. There are, however, a few common-sense reflections, of a purely rational kind, which first claim our attention.

Art. 2. The Velocity V.—In speaking of the steady flow of water in a pipe, or in an open channel of prismatic form, it would almost seem that some of the earlier writers must have conceived the motion to be something like the sliding of a block of ice. If it were true that all the particles of water in such a current do indeed move together in straight parallel lines, and at the same speed, the problem would be greatly simplified, and we should understand the velocity V to indicate the common velocity of all the particles.

A little observation of nature is sufficient to show that no such motion takes place in any visible stream. Looking at the surface of the current as it flows under a bridge, we notice at once that the velocity is greater at the centre than at the banks of the stream. Further experiment shows very readily that the velocity is greater at the surface than at the bottom of the river; and in the case of

a cylindrical pipe, it has been ascertained that the linear velocity is greatest at the axis of the pipe, and least at the circumference.

Nevertheless, if we take a normal cross-section of the current at any point in its course, we may understand the symbol V to indicate the mean velocity of all the filaments which traverse that cross-section.

Thus if a denotes the sectional area of the pipe or of the prismatic channel in square feet, and if V is the mean velocity in feet per second, then the quantity of water which passes the given cross-section in every second, or the discharge in cubic feet per second, will be—

$$Q = Va \quad . \quad . \quad . \qquad . \quad (1)$$

This simple expression will be equally true whether the water moves like a block of ice, or whether the different filaments pass the cross-section at different speeds; and it may be well to notice some of the facts which it implies.

Imagine, first, that some natural watercourse, following a sinuous line along the valley, flows in a bed of irregular form, sometimes deep, and sometimes shallow; but suppose that, along a given stretch of some miles, the river receives no tributary, and loses none of its water by diversions or leakage. The quantity Q is constant along the whole of that stretch, and at each point in its course V is inversely proportional to a. Wherever the bed is narrow and shallow the velocity will be great, and where the channel has a large section the current will be slow. Still waters run deep. And at every point where the sectional area is seen to change, the flow of water must undergo a *change* of velocity—an acceleration, or a retardation of its speed.

Suppose, next, that the current flows in some straight prismatic channel of perfectly regular (rectangular) form, and without receiving or losing any water through the sides of the channel. Then Q will again be a constant quantity, and at each cross-section the mean velocity V will depend upon the height at which the water-surface stands above the floor. If the depth of water is greater at the upper end of the stream than at the lower end, it will show that the flow is undergoing an acceleration in its course along the channel—and *vice versâ*. But if a longitudinal section of the stream shows that the water-surface is exactly parallel with the floor, then V is a constant quantity, and the current, as a whole, undergoes neither acceleration nor retardation, during its course.

Lastly, take the case of a long line of cylindrical pipe, whose joints are all water-tight. When the pipe is running full Q must have the same value at all cross-sections, for it is obviously impossible that the quantity which leaves the pipe at one end can be either greater or less than the quantity which simultaneously enters at the other end. At the same time we have, in this particular case, a sectional area a which is absolutely uniform from one end of the line to the other. Therefore V must be constant for all cross-sections: that is to say, at any given instant of time, the velocity V is the same at all points in the line. To increase or diminish that instantaneous velocity, we should have to deal with the inertia of the whole column of water filling the pipe.

In great works of water-supply, the gravitation main which conveys the water from one reservoir to another may have an unbroken length of many miles. When such a main, however, has been filled with water throughout, and brought to its regular working conditions, the flow will gradually attain to a steady "regime;" and when this has been established, the discharge goes on without any alteration from minute to minute, or from hour to hour. In such a case as this we have an example of a very uniform flow. It is not the motion of a block of ice, for the central particles have a greater linear velocity than those which are moving close to the sides of the pipe. But the flow is uniform in two senses. The velocity V is exactly the same at all cross-sections of the pipe from one end to the other; and at each cross-section the velocity remains constant as time goes on.

Art 3. The Stages of the Journey.—The uniform flow of water which has just been described as taking place in a long cylindrical pipe, reminds us that there are yet some other questions which must be borne in mind when we are considering any current whose discharge Q is constant.

Setting aside, for the present, any question as to the relative velocities of the different filaments, the mean velocity V undergoes no change whatever during the journey from one end of the pipe to the other; and the same thing may be seen every day in open channels of uniform section and uniform depth: but however long may be the journey through the conduit, it must have a beginning and an end.

At the upper end of the pipe or conduit, the water enters the inlet from some reservoir, which we will suppose to be a large

one, and in which the fluid is at rest, or very nearly at rest. And at the outlet it is discharged into another reservoir, where its journey comes to an end, and it returns either to a state of rest, or nearly so, after producing some gurgitation in the quiet water. The journey has at least three stages which require to be separately considered—the start, the voyage through the conduit, and the finish.

At the first starting of the current, energy must be expended in the work of acceleration. The inertia of the water must be overcome: the particles are lying at rest, and they must be made to move with the velocity V (or with the average velocity V), at the moment they enter the cylindrical bore of the pipe. In all natural watercourses, canals, or gravitation mains, the force by which this accelerating effect is produced is always the force of gravity. In other words, the energy by which the work of acceleration is performed, is the energy derived from the fall of the water through a certain vertical height, or from a certain "loss of head;" and the ordinary principles of dynamics should be sufficient to determine the velocity that would be produced by any given fall, or to calculate the fall or loss of head h that is requisite for the first production of any given velocity. Experiment has shown, indeed, that at this point of entry into the pipe-inlet, the water starts off in accordance with those principles—or very nearly so.

But when the water has once entered such a conduit of uniform section, the second stage of its journey appears to be performed in a very different manner; for the velocity V undergoes no change from this point onwards. Energy is still expended, but expended solely in *maintaining* the motion of the uniform speed V, and without producing any further acceleration. This is clearly to be seen in any channel of uniform section in which the stream follows its downward course upon a continuous gradient; for the descent of the water upon this inclined plane implies a continuous expenditure of energy, and therefore the continuous performance of some kind of work, which apparently is *not* the work of acceleration.

If the steadily moving column had been a column of ice, we should probably have accounted for its uniform motion by supposing that the inclination of the plane had been adjusted to the "angle of repose," so that the energy of the descent was exactly expended in overcoming the frictional resistance—leaving no excess or deficiency that could produce an increase or decrease of speed.

On such a table, the ice, having once been started with the velocity V, would continue to slide down for any distance without changing that velocity.

The uniform flow of water has sometimes been compared to a case like this, regarding it as motion against some kind of frictional resistance which takes effect at the bottom and at the sides of the channel. The analogy is not complete, although it has contributed something towards the construction of the formula which is commonly used to-day.

But whatever may be the true character of the work that is here done, it is certainly done by the action of gravity, *i.e.* by the descent of the water from a higher to a lower level, through some vertical height, h_2, which will evidently be proportional to the length of the canal. And this fall h_2 is something quite distinct from the first fall h_1, by which the water acquired the velocity V at the start.

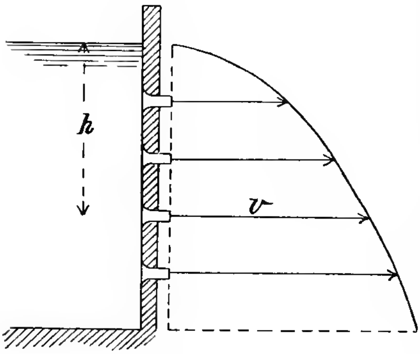
For the chief purposes of the hydraulic engineer, the calculations of flow and discharge require, in almost every case, that these two losses of head, h_1 and h_2, should be separately computed; and in many cases their sum, $h_1 + h_2$, may be taken as being practically equivalent to the total fall H from start to finish, or at all events from the starting-point to the outlet of the conduit.

At the outlet there will be some further questions which will depend upon the circumstances of each individual case; but leaving these for the present, it may be well to notice that, if the fall H is measured from start to outlet, it must always be considered as consisting of the two parts h_1 and h_2, whose relative magnitudes have to be determined. They cannot be taken together, because they represent the losses of head due to the performance of two different classes of work, and because their relative magnitudes will vary enormously in different cases. The fall h_2 is proportional to the length of the conduit, which may be very great in some cases and indefinitely small in others, while the fall h_1 is independent of the length, and is determined only by the velocity V

Thus, in a long water-main the loss h_1 may be less than the one-thousandth part of the whole fall, while in a shorter pipe or culvert it may amount to more than one-half; and when the water flows through a submerged sluice-opening it constitutes practically the whole of the fall, for in such a case the length of conduit is almost nothing—the start and the finish comprise the whole journey.

Art. 4. The Work of Acceleration.—The principles which serve to determine theoretically the first loss of head, h_1, are fairly well illustrated by the free discharge of water through an orifice. If a jet of water is allowed to issue horizontally through one of the orifices sketched in Fig. 1, discharging itself into the open air without encountering any back pressure or any serious frictional resistance, we may almost assume that the only work to be done in this case consists in overcoming the inertia of the particles, and thus generating the velocity with which they are seen to issue from the orifice; or, in other words, that the energy expended is almost wholly transformed into the kinetic energy of the moving stream.

FIG. 1



If the centre of the orifice is at the vertical depth h below the water-surface in the reservoir, and if the reservoir is constantly fed at top-water level while the same quantity Q is drawn off per second through the orifice, it is evident that the constant expenditure of energy per second will be $Q\gamma h$, where γ is the weight per cubic foot, and $Q\gamma$ the weight of water discharged per second. Thus, if $h = 1$ foot, every pound of water that is drawn off through the orifice loses one foot-pound of its potential energy by its descent from a higher to a lower level.

At the same time, if v is the velocity of the issuing jet, the kinetic energy of the body of water, whose weight is $Q\gamma$, will be given by the well-known expression, $Q\gamma\frac{v^2}{2g}$, where g is the gravity symbol. Therefore, if the energy of the fall were wholly expended in the work of acceleration, we should have—

$$Q\gamma h = Q\gamma\frac{v^2}{2g}$$

$$\text{whence } h = \frac{v^2}{2g} = \frac{v^2}{64{\cdot}4} \quad . \quad . \quad . \quad (2)$$

$$\left.\begin{aligned}\text{or } v = \sqrt{2gh} &= \sqrt{64{\cdot}4h} \\ &= 8\sqrt{h}\text{ nearly}\end{aligned}\right\} \quad . \quad . \quad (2a)$$

And this means, of course, that the particles would issue with the same velocity v that would be acquired by any solid body in falling vertically through the height h; for the free fall of such a body *in vacuo* is the perfect and typical example of energy spent wholly in the work of acceleration.

Experiment shows that this theory can be applied without much error if due regard is paid to the conditions which it implies. An exact agreement is hardly to be expected in any case, for it is not certain, nor indeed probable, that all the particles move at exactly the same speed at the moment of issue, and it is certain that some small amount of energy is expended in frictional work.

The experiment is often made by gauging the actual discharge, Q, from an orifice of known dimensions under a given head h, or under a series of different heads.

The mean velocity V must then be ascertained by the expression, $V = \frac{Q}{a}$, where a is the sectional area of the jet at the point where acceleration is completed.

If the water is drawn off through a short conoidal mouthpiece, entering it from the trumpet-shaped end, the jet is discharged from the small end in a nearly cylindrical stream; and the sectional area a of this stream will be the same thing as the sectional area a_0 of the orifice. The mean velocity of the particles, as they issue from a properly shaped mouthpiece of this kind, is found to differ but little from the theoretical velocity due to the head h, ranging in fact from 96 to 99, or perhaps $99\frac{1}{2}$ per cent. of that quantity; and we may write $V = \frac{Q}{a} = \frac{Q}{a_0} = \sqrt{2gh} \times \phi$, where ϕ is a coefficient varying from 0·96 to 0·994.

If the circular orifice is formed in a thin plate, countersunk on the outer side so as to present nothing but a sharp edge to the stream flowing through it, experiment shows very distinctly that the particles do not cross the plane of this orifice in parallel lines; for at first the stream converges rapidly, just as though the particles were entering the wide end of a trumpet-shaped mouthpiece. But, at a very short distance outside the plate, the jet assumes a truly cylindrical form; and, so far as can be seen, the particles appear to move onwards at uniform speed, in truly parallel lines, and without any sign of internal disturbance.

Along this portion of its course, the jet resembles, in outward appearance, a cylindrical rod of polished glass—its sectional area a

remains sensibly constant for some distance, and therefore also its mean velocity V. But this velocity is considerably greater—almost 50 per cent. greater—than the linear velocity V_0 at the plane of the orifice as computed from the expression $\frac{Q}{a_0}$, because the diameter of the cylindrical stream is considerably less than that of the orifice. When the contraction is perfect, the sectional area a is about 0·64 a_0, and therefore the velocity V_0 at the plane of the orifice is only about 0·64 V. The complete effect of the work of acceleration is therefore not to be measured at the plane of the orifice, where some of the particles are moving obliquely, and where the stream as a whole has not yet attained its full speed; but if we measure the velocity V at the cylindrical part of the jet, we shall again find that experiment agrees very nearly with theory. On the average of a large number of gaugings, the observed value of V appears to be about 97 per cent. of the velocity which should theoretically be generated by the head h, so that again we might write $V = \frac{Q}{a} = \sqrt{2gh} \times \phi$, taking ϕ at the mean value 0·97 or thereabout. Of course this expression will be quite useless for the purpose of calculating the discharge through any orifice of known sectional area a_0, unless we have ascertained the true value of the ratio $\frac{a}{a_0}$, or "coefficient of contraction," as it is usually called, for the theoretical velocity V applies only to the contracted stream, whose sectional area a is not always known beforehand, and may often be much smaller than the known area a_0 of the orifice.

In the two examples already mentioned, where the ratio $\frac{a}{a_0}$ was equal to unity in the one case, and only 0·64 in the other, the velocity observed *in the contracted stream* was nearly the same in each; and in both experiments it is found that the "coefficient of velocity" ϕ undergoes very little change with varying values of the head h. That is to say, whether the head be very small or very great, V is nearly proportional to $\sqrt{h}$, so that the relative velocities of different streams issuing from a series of orifices, one below the other, might be represented, as in Fig. 1, by the ordinates to a parabolic curve, whose vertex coincides with the top-water level in the reservoir.

Art. 5. Successive Changes of Speed.—The parabolic diagram

may serve to keep before us those relations which, theoretically, should be found to subsist between V and h during any acceleration of the stream, and which apparently *do* subsist, pretty nearly, when the experiment is made under suitable conditions. Wherever a fall h is expended in starting the particles of water into motion from a condition of rest, we may expect to get (in the contracted stream) a velocity approximating to $8\sqrt{h}$, though it may not quite reach that value. For a fall of 1 foot the velocity will be nearly 8 feet per second; for a fall of 2 feet it will be nearly $8\sqrt{2}$, and so on. As h is augmented by a succession of equal increments, the diagram shows that V will be augmented by increments which form a series of continually diminishing quantities.

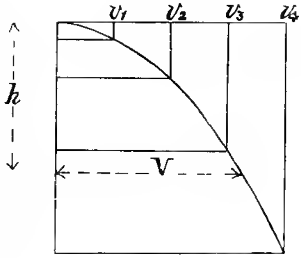
In the same way, wherever we find the stream running with the velocity V, we may conclude that the generation of that velocity has entailed a loss of head proportional to V^2, and amounting theoretically to $\frac{V^2}{64}$. Therefore, if the stream at a certain point in its course, is running with the velocity V_1, and a little further on with a greater velocity V_2, we may conclude that, in the interval between those points, there has been a further loss of head, which may be expressed by—

$$h_2 - h_1 = \frac{V_2^2 - V_1^2}{2g} \qquad (3)$$

or, $h_2 - h_1 = \frac{1}{64}(V_2^2 - V_1^2) = \frac{1}{64}(V_2 - V_1)(V_2 + V_1)$ nearly.

It is worth while to notice that the additional fall which must take place at this point to account for the increased velocity is not to be measured merely in terms of the increment of velocity, but is proportional to that increment multiplied by twice the mean velocity. Thus the diagram, as sketched in Fig. 2, shows that if V is to be augmented by a succession of equal increments, the fall h must be augmented by increments which form a series of continually increasing quantities.

FIG. 2



EXAMPLES.—If V = 1 foot per second, $h = \frac{1}{64} = 0{\cdot}016$ feet; and if V = 10 feet per second, $h = \frac{10^2}{64} = 1{\cdot}61$ feet.

To increase the velocity from 10 feet per second to 11 feet per second, the additional loss of head would be $h_2 - h_1 = \frac{11^2 - 10^2}{64} = 0{\cdot}328$ feet.

To increase the velocity from 20 feet per second to 21 feet per second, the additional loss of head required for the same increment of velocity would be $h_2 - h_1 = \frac{21^2 - 20^2}{64} = 0{\cdot}64$ feet.

Art. 6. The Retardation of the Stream.—If the acceleration of the stream is to be treated as a question in dynamics, we must pursue the application of the same principles a little further, and consider the question of bringing the water to rest at the end of its journey.

Every pound of water moving with the velocity v possesses the kinetic energy $\frac{v}{2g}$ (in foot-pounds); and it cannot be brought to rest until that energy has been expended in the performance of work against *some* kind of resistance.

In applied mechanics we are familiar with a number of examples, such as the fly-wheel and the pendulum, where the energy stored up during acceleration is restored again during the retardation of the body; and if the moving particles of water could be made to expend their kinetic energy in raising themselves against the force of gravity, the same result would ensue—the loss of head due to acceleration at the start should be capable of recovery at the end of the journey.

It is, indeed, quite easy to produce this result, or something very much like it, by simply turning the jet in a direction vertically upwards, as sketched in Fig. 3, when the jet would rise almost to top-water level if there were no frictional resistance in its passage through the mouthpiece and through the air.

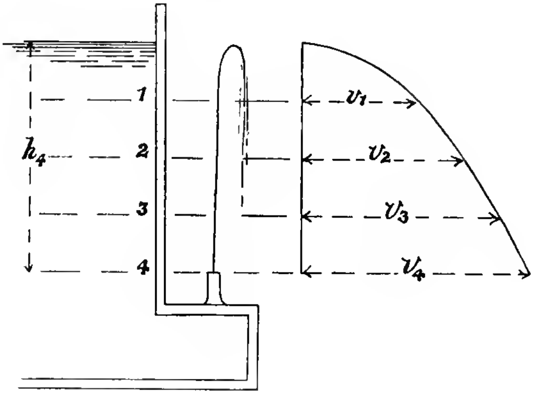
As the water is raised to this height, through the successive stages indicated on the sketch, the kinetic energy of the stream is gradually spent in the work of raising it from stage to stage, and thus restoring the potential energy which it had possessed at the higher level.

To illustrate this transformation of energy, suppose the jet to issue from the conoidal mouthpiece at a depth $h_4 = 4$ feet below top-water level, with the velocity $v_4 = 8\sqrt{4} = 16$ feet per second. At this point the kinetic energy possessed by each pound of the moving water is $\frac{16^2}{64} = 4$ foot-pounds; but in rising through each successive foot of height, one foot-pound of work is performed, and the kinetic energy is consequently reduced by that amount. At any height x above the nozzle, the kinetic energy for

each pound is simply $h_4 - x$, and is measured directly by the instantaneous depth h below top-water: while the velocity of the ascending particles is everywhere proportional to $\sqrt{h}$, and at each point may be represented by the corresponding ordinate in the parabolic curve.

The ascent comes to its termination when $x = h_4$, or $h = 0$, and the water is now for an instant at rest; so that if the stream were caught at this point, the journey would have been performed with-

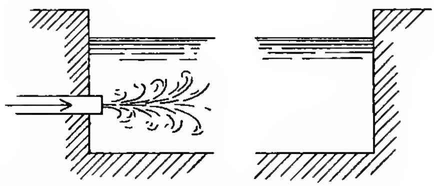
FIG. 3



out any loss of head except that which might be due to small frictional resistances—the acceleration at the start would have cost nothing, because the work of acceleration would have been wholly restored.

How far it may be possible to reproduce this kind of retardation in the flow through a discharging outlet, so as to recover some part of the head lost in acceleration, is a question that will have to be considered later on; but experiment shows that no such recovery takes place when the stream is discharged through a submerged cylindrical outlet into the quiet waters of a reservoir.

FIG. 4



In any such case as that sketched in Fig. 4, the kinetic energy of the penetrating stream appears to be wholly spent in producing

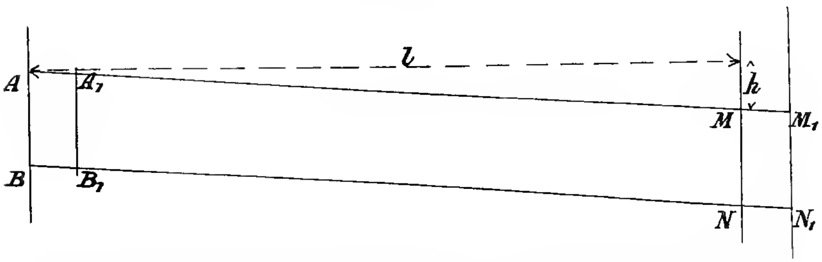
eddies or gurgitations of some kind in the surrounding mass of fluid; and energy spent in this way appears to be wholly lost for the practical purposes of the engineer.

Such examples show very clearly that when water travels through water, its progress is quickly arrested by a sort of frictional grip. When the jet issues from a well-formed orifice in a thin plate, it travels through air with very little resistance, and its polished surface shows no sign of a ruffle for a considerable distance; but when it enters a body of quiet water it tends to carry along with it the surrounding contiguous particles, which in their turn tend to arrest the progress of the jet, or to carry along with them the next adjacent particles on the quieter side. Thus a rotary motion seems to be set up by the action and reaction on opposite sides of a particle or layer; and the kinetic energy is quickly dissipated in the eddies which are produced on all sides.

Art. 7. Uniform Flow in a Prismatic Channel.—Between the commencement of the journey in one reservoir and its termination in another, we have to consider the progress of the stream along the conduit, and we will first suppose that the intervening conduit is an open channel of prismatic form.

Let Fig. 5 represent the longitudinal section of the channel for a certain length l intercepted between two sections AB and MN; the channel being laid upon a uniform gradient throughout its

FIG. 5



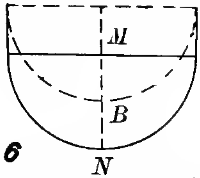
entire course, which may be supposed to extend indefinitely beyond these sections in both directions. Also let h be the vertical fall on the length l, so that the bed of the channel BN has the uniform slope $s = \frac{h}{l}$; and suppose the water-surface AM to be exactly parallel to the bed BN.

The flow of a stream may often be seen to fulfil these conditions very nearly, the sectional area a being sensibly constant

for a considerable stretch of its course; and it follows that, during the whole of its progress along such a reach, the velocity V undergoes no acceleration and no retardation.

Nevertheless it is obvious that, during the continuous fall of the stream, energy is expended in the performance of some kind of work at every step of the way. After traversing the length l, every particle or element in the sectional plane MN has come to occupy a position lower than that of the corresponding particle in the plane AB—every particle has, in fact, fallen through the vertical height h; and the work accomplished by this fall has been performed with uniform regularity during the transit, as though it were done against some unvarying resistance; for each foot of the journey has been accompanied by the same fall s, which represents the fraction $\frac{h}{l}$, or the fall of the stream per foot of its course, and therefore the same work has been done on each successive foot of the forward motion.

FIG. 6



If we could believe that all the particles move together with the same velocity V, we should be driven to the conclusion that the resistance above mentioned must be due entirely to the friction between the water and the bed of the channel or its walls.

When the prism $ABMN$ moves forward on the incline to a new position $A_1B_1M_1N_1$ through any distance $AA_1 = x$, it falls through the vertical height sx, while its weight will be simply $al\gamma$ (where γ is the weight per cubic foot and a the sectional area of the prism), so that the energy expended will be $al\gamma sx$. And if this movement were effected against a constant frictional resistance F, the work done would be Fx, and the resistance would be—

$$\text{F} = al\gamma s \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

which is just the weight of the prism $al\gamma$ multiplied by the ratio $\frac{h}{l} = s$.

This formula must, indeed, express the true *value* of the resistance or retarding force, whatever may be its true nature; but if it is regarded as friction upon the channel-bed, then the friction of fluid upon solid must be something very different from the friction between solid substances. For if the co-efficient of that friction were independent of the velocity it is obvious that the

water would not flow at all unless the channel were tilted to a certain inclination corresponding with the angle of repose. If the channel were laid at any inclination less than the "angle of repose" the water would stand on the slope, and at any greater inclination it would run down the channel with a constantly accelerating speed, but neither of these conclusions is warranted by the facts. The most commonplace observations are enough to show that on a gentle inclination we have a uniform flow at slow velocity, and in more steeply inclined channels we have a flow which is still uniform, but at greater speed. This would be impossible unless the resistance varied with some function of the velocity V.

Of course the actual value of the supposed frictional resistance could only be found by experiment; and from very early experiments the following results were deduced:—

1. The friction is quite independent of the fluid pressure per unit of area, and does not, therefore, vary with the depth of water or hydrostatic head over each element of the wall-surface.

2. It is nearly proportional to the superficial area of the surface of contact: *i.e.* F is proportional to lp, where p is the "wetted perimeter" of the prism, measured round the cross-section:—and this rule seemed to hold good for channels having various forms of cross-section.

3. It is approximately proportional to the squares of the velocity, or V^2.

Hence it appeared that, in general, F was proportional to lpV^2.

If these deductions were correct, and if it were found experimentally that F was equal to lpV^2 multiplied by $A\gamma$, where A is some co-efficient determined by the experiment, it would be quite a simple matter to write, generally—

$$al\gamma sx = Fx = A\gamma lpV^2x$$

(equating the energy expended with the work performed), and we should immediately have—

$$as = ApV^2, \text{ or } s = AV^2 \cdot \frac{p}{a}$$

This expression would determine the inclination s that must be given to any canal in order to maintain a certain constant velocity V. It would be simply proportional to V^2, and also proportional to the ratio $\frac{p}{a}$.

Conversely, when we know the slope s, the sectional area a, and the wetted perimeter p, the velocity of the current would be found by the expression—

$$V^2 = \frac{s}{A} \times \frac{a}{p}$$

The quantity $\frac{a}{p}$ is, of course, a linear dimension, which is easily calculated for a canal of any given cross-section. By English writers the dimension has been called the "hydraulic mean depth," and in the case of a stream of indefinite width running over a flat bottom, it is, of course, the same thing as the "depth" of the water: but for brevity's sake, modern writers have preferred to call it the "hydraulic radius," although it is not the radius of any known geometrical figure. However, it is commonly designated by the symbol R, and, using this letter to denote the quantity $\frac{a}{p}$, the formula can be shortened up, and becomes—

$$s = \frac{AV^2}{R}\,; \text{ or, } V^2 = \frac{Rs}{A}\,; \text{ or, } V = \sqrt{\frac{Rs}{A}} \qquad . \quad (5)$$

while the total fall on any length l will be—

$$h = sl = \frac{AlV^2}{R}$$

This is the very important formula, which, in one shape or another, is used all the world over for the determination of a multitude of questions connected with the uniform flow of water. It appeals, as we have seen, to some rational or quasi-rational principles; but it takes no notice of any effects that may be due to the roughness or smoothness of the bed, and it depends upon the empirical inferences which were drawn from a few experiments.

As we shall have to examine these assumptions in the light of later experiments, it may be well to notice the various shapes which the formula takes in the writings of different observers.

The coefficient A, which has been employed in formula (5) as an experimental measure of the resistance, is usually adopted in the works of D'Arcy, Bazin, and some other French observers; but the quantity A is always a very small decimal fraction, and it is perhaps more convenient to use its reciprocal, which is always a large whole number. Thus, if we make $K = \frac{1}{A}$, we shall have the

formula in a shape which has been more customary with English hydraulicians, viz.—

$$s = \frac{V^2}{KR}; \text{ or } V^2 = KRs; \text{ or } V = \sqrt{KRs} \quad . \qquad (6)$$

$$\text{and the total fall will be } h = \frac{V^2}{K} \cdot \frac{l}{R}.$$

Again, if we make $c = \sqrt{K}$, the formula will take the shape which is adopted in many American text-books, *i.e.*—

$$s = \frac{V^2}{Rc^2}; \text{ or } V^2 = Rsc^2; \text{ or } V = c\sqrt{Rs}. \qquad (7)$$

$$\text{while } h = \frac{V^2}{c^2} \cdot \frac{l}{R}$$

Once more the formula takes a different shape when s and h are expressed in terms of the kinetic energy $\frac{V^2}{2g}$, as in the writings of Weisbach and many other hydraulicians. Thus, making $\zeta = A \times 2g$, we obtain—

$$s = \frac{V^2}{2g} \cdot \zeta \cdot \frac{1}{R}; \text{ or } V^2 = 2gRs\frac{1}{\zeta}; \text{ or } V = \sqrt{2gRs\frac{1}{\zeta}} \quad (8)$$

$$\text{so that } h = \frac{V^2}{2g} \cdot \zeta \cdot \frac{l}{R}$$

Of course all these expressions come to exactly the same thing. The coefficient A, K, c, or ζ, as the case may be, is only to be found by experiment, and its value has been calculated from a number of different observations by means of the inverted formulæ given in the second column of the following table, in which s and R represent the known inclination and the hydraulic radius, while V represents the observed velocity of the stream.

TABLE 1.—EQUIVALENT VALUES OF THE COEFFICIENT.

Formula.	Coefficient deduced by experiment.	Equivalent Values.
(5) $V = \sqrt{\frac{Rs}{A}}$	$A = \frac{Rs}{V^2}$	$A = \frac{1}{K} = \frac{1}{c^2} = \frac{\zeta}{2g}$
(6) $V = \sqrt{KRs}$	$K = \frac{V^2}{Rs}$	$K = \frac{1}{A} = c^2 = \frac{2g}{\zeta}$
(7) $V = c\sqrt{Rs}$	$c = \frac{V^2}{\sqrt{Rs}}$	$c = \sqrt{K} = \frac{1}{\sqrt{A}} = \sqrt{\frac{2g}{\zeta}}$
(8) $V = \sqrt{\frac{2gRs}{\zeta}}$	$\zeta = 2g \cdot \frac{Rs}{V^2}$	$\zeta = \frac{2g}{K} = \frac{2g}{c^2} = 2gA$

If the formula were correct and generally applicable, the coefficient, as determined in this way from any number of different experiments, should have a constant value; but the experiments prove that the coefficient is *not* constant. To make the formula fit in with the facts, the coefficient must be taken at widely different values in different cases. And yet the results of our practical calculations will obviously depend upon the value assumed for the coefficient—the calculated discharge will be exactly proportional to the value assumed for c; the calculated fall h will be proportional to the value assumed for A.

The most notable divergencies are those which have been observed under the following conditions:—

1. Natural watercourses, which are often very irregular in form, and sometimes blocked with weeds, can hardly be expected to conform very closely with the rule—or, perhaps, with *any* rule. Mr. Trautwine's list of 282 gaugings, in different rivers, shows that the coefficient c varies from about 20 to nearly 200; so that K varies from 400 to something like 40,000. But the very high values are only obtained in the largest rivers, while the very low ones are confined to quite shallow streams. A further list of 300 gaugings, taken in smaller streams, and in canals of more regular form, gives values of c ranging from 20 to 112; and again the lowest values are obtained either in shallow channels or in streams with very rough beds.

Such deviations were always to be expected in irregular streams, but it was at first believed that the coefficient would be nearly constant in all artificial channels of regular prismatic form, with a fairly smooth lining. For, in all such cases, the older gaugings yielded results which were tolerably consistent; and they were accepted as indicating that the coefficient was not much affected by a little roughness in the lining, and was equally correct for channels of all sizes, and of all ordinary forms of cross-section; but this assumption has proved to be a mistaken one.

2. The more complete experiments of M. Bazin have proved that the flow is *greatly* affected by the roughness or smoothness of the walls. When a lining of rubble masonry is exchanged for a smooth lining of neat cement, the discharge is almost doubled in value if the channel is a small one.[1] The *extent* of the gain,

[1] This seems to justify the practice of the ancient Romans, who were at great pains to polish the internal surfaces of their aqueducts.

however, appears to depend upon the size of the channel. In large conduits it is not so great as in small ones.

3. Among conduits which are all lined with *the same material*, it is found that the coefficient is not independent of the size of the conduit. In channels of semicircular section it varies with every change of the diameter; and in all forms of cross-section it varies with every change in the hydraulic radius R. The law which governs this variation is rather obscure, but it becomes an important question when the probable discharge of a large conduit has to be calculated from experiments made on a smaller scale.

4. It is not quite true, either, that the resistance is to be accurately measured by the length of the wetted perimeter p, without any regard to the *form* of cross-section. Of course the theory, as it stands, *implies* that a semicircular channel would have a certain advantage over any other channel of the same area a, because its wetted perimeter p is the smallest; but in practice the advantage of the semicircular form is somewhat *greater* than the theory indicates. With this exception, however, the coefficient shows but little variation in all the ordinary forms of cross-section.

5. Among a number of channels which are all built to the same section, and lined with the same material, but laid at different inclinations, the coefficient c is not quite constant. With the greater slope s, the coefficient is persistently greater throughout one large group of experiments, and persistently less throughout another large group; while in a great many cases of ordinary occurrence the variation is exceedingly small. By some writers the variation is attributed to change in the velocity V, while others prefer to connect it with the change of slope.

All these points must be examined in detail if the facts are to be rightly construed; and they must be taken account of, in some way, if our calculations are to be nearly correct. To bring the facts properly into account, numerous methods have been proposed, and the formulæ that have been devised for this purpose may be classed under two heads :—

1. Those which proceed by a reconstruction of the fundamental formula, introducing some different functions of V, R, and s.

2. Those which leave the old rule as it stands, employing a coefficient which will vary in different cases, and which is to be determined by means of another empirical formula.

The two methods are equally empirical in character, and, on

the score of convenience, the engineer may perhaps prefer the latter when he is dealing with the numerous calculations that are involved in the design of aqueducts, canals, and culverts presenting various forms of cross-section.

Art. 8. Uniform Flow in Pipes.—For a circular cross-section the area a and the wetted perimeter p are exactly twice as great as in a semicircular channel of the same diameter d, so that in both cases R has the same value, and is equal to $\frac{d}{4}$. Hence it is generally assumed that, in a circular pipe or culvert laid upon any uniform gradient, the velocity V will be just the same when the pipe is running full, as when it is running half-full, and that the discharge of the full pipe will be exactly twice the discharge of the semicircular channel.

Some experiments which have been made by M. Bazin with a square wooden trunk show, indeed, that, when the trunk is running full, the *relative* velocities of the different particles are not quite the same as they are in the lower half of the trunk when it is used as an open channel; but whether the trunk was running full or exactly half-full—the hydraulic radius being the same—the *mean* velocity V was found to be very nearly the same. From which M. Bazin concludes that the stream in an open channel does not experience any considerable resistance from the air at its upper surface, and that the wall-resistance is nearly the same in pipes and in channels.

At the same time it will be remembered that this skin-friction is quite independent of the fluid-pressure, so that a pipe running full at any given velocity V should exhibit the same resistance, and the same loss of head per mile, whatever may be the pressure under which it is charged.

Thus, if an inverted syphon were laid from one reservoir to another, across a deep valley, as sketched in Fig. 7, the skin-friction would be the same at all parts of its course, irrespective of the depth of the valley. And the loss of head per mile, or the total loss of head due to this skin-friction, would be the same as though the pipe were laid in one continuous gradient from reservoir to reservoir; or as though the lower semicircular half of it were laid as an open channel, and the fall were measured by the inclination of the visible water-surface. All this is clearly illustrated, and sufficiently well confirmed by the results of numerous "piezometer" experiments.

If a number of vertical stand-pipes were attached to the main at various points along its course, the water would rise in each of them to a certain height, equilibrating the effective pressure in the main at that point; and when the flow is uniform, the water-levels in all the stand-pipes are found to lie in a nearly straight inclined line A, B, C, D, E—provided that the syphon is of uniform diameter throughout, so that the velocity is uniform. Thus, if all the stand-pipes are fixed at the same distance apart (measured along the pipe), there will be the same loss of head between each

FIG. 7

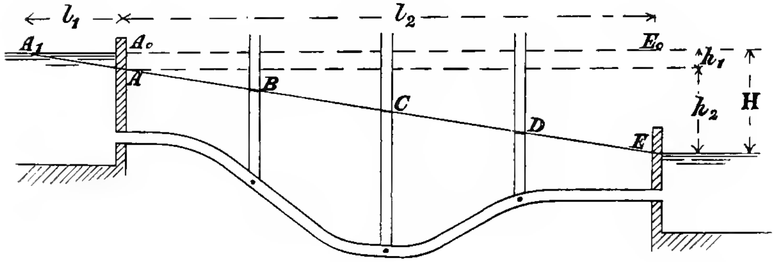
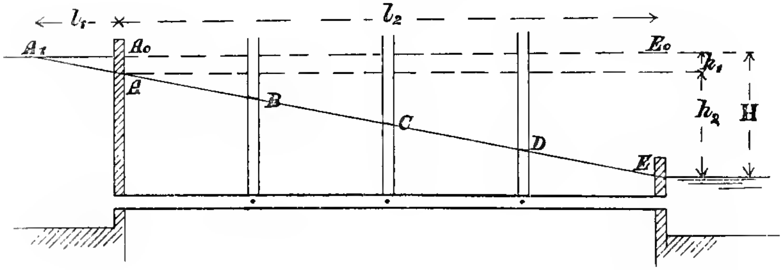


FIG. 7A



pair, no matter what may be the contour of the valley, or the ups and downs of the pipe—so long as the pipe does not rise anywhere above the inclined line AE.

The piezometer readings, whether they are taken in an actual stand-pipe, or by any kind of pressure-gauge, will undoubtedly be affected by the manner in which the stand-pipe, or gauge, is connected to the main; but the loss of head between any two stand-pipes will, nevertheless, be correctly measured if both connections are made in *the same* way—as, for example, by an orifice

entering the main horizontally on one side, and at right angles to its axis.

Such observations have shown conclusively that the resistance is unaffected by the pressure, however great that pressure may be. With a giveu velocity V, the loss of head h_2 between the points A and E is simply proportional to the length of pipe l_2, just as it is in open channels; and we may use the same formula in both cases to determine the loss of head for any given velocity, or to find the velocity when the fall is known. The symbol s in the old formula may be used to express the ratio $\frac{h_2}{l_2}$, or the inclination of the "hydraulic mean gradient."

To measure the gradient quite accurately, the length l_2 should certainly be measured *along* the *pipe*, following its ups and downs; but in many practical cases we can afford to neglect the slight difference between the horizontal length and the actual slope-length; and then the hydraulic mean gradient becomes a truly straight line, which can be drawn at once upon the longitudinal section, from one reservoir to another.

This line upon the section is of the utmost importance to the hydraulic engineer, and will be used by him for an immense number of calculations, all along the route.

In drawing the gradient upon the longitudinal section, the lower end can generally be fixed without much difficulty when the water is to be freely discharged at the outlet without any choking by a stop-valve. If the flow is delivered into the air from the open mouth of the pipe, falling perhaps into a basin below, the end of the hydraulic gradient at E will coincide with the centre of the pipe. And when the cylindrical outlet is submerged beneath the quiet waters of the low-level reservoir, as sketched in Fig. 7 and in Fig. 4, the end of the gradient at E will coincide with the water-level in that reservoir.

But at the upper end A, the gradient never coincides with the water-surface in the high-level reservoir. At this point the experiments always show that there is a certain drop $A_0 \ldots A$ at the very commencement of the journey, and it is, of course, none other than the loss of head h_1 which is expended in the work of acceleration at the moment when the particles are started from a condition of rest.

Hence the total fall H, from one reservoir to another, must be regarded as consisting of two items h_1 and h_2, of which the former

is nearly equal to, or, at least, proportional to, $\frac{V^2}{2g}$, and independent of the length of the main, while the latter is proportional to the length l_2.

It will be seen, therefore, that the true slope s is always something different from the ratio $\frac{H}{l_2}$; and when the conduit happens to be a short one, it will be very widely different from that ratio; so that it is quite impossible to construct any correct Table of Discharges in terms of the quantities H and l_2, which are the quantities generally known beforehand, or in terms of the inclination $\frac{H}{l_2}$.

With this modified definition of the gradient s, we might use the old formula $V = \sqrt{KRs}$ or $V = c\sqrt{Rs}$ to calculate the velocity in pipes, or in circular culverts running full—remembering that R is always one-fourth of the diameter, or $\frac{d}{4}$. And the calculation will come to the same thing if we express V in terms of d, using for the circular section a special value for the coefficient, as many writers have done. Thus, if we make $K_p = \frac{1}{4}K$, and $c_p = \frac{1}{2}c$, we might write $V = \sqrt{K_p ds}$, or $V = c_p\sqrt{ds}$.

In every case the coefficient must be found by experiment, and the correctness of the formula must be tested by observing its constancy or its aberrations in different experiments.

A comparison of such gaugings in circular pipes shows in general that—

1. The regularity of form prevents any such wild aberrations as are to be noticed in the gaugings of natural watercourses.

2. In pipes of *moderate dimensions*, the resistance is very greatly affected by a little roughness—so that sometimes a very slight accumulation of rust on the surface is enough to reduce the coefficient c by one-half—or, in other words, to increase the resistance fourfold.

3. The coefficient increases with the diameter according to some law which it is important to discover.

4. The coefficient is not quite the same at all velocities. In a few gaugings it seems to vary inversely with V, but in the great majority of cases it increases persistently as the velocity increases.

These remarks would apply to *most* experiments that have been made under *ordinary* conditions; but it is worthy of special notice, that when the diameter is very small and the stream very

slow, the resistance appears to follow a quite different law, and is very nearly proportional to V instead of V^2.

Art 9. The Resistance to Uniform Flow.—It has already been mentioned that, when the water travels along a pipe or conduit without changing its mean velocity V, the flow is only to be regarded as uniform in the sense that the water *as a whole* undergoes no acceleration and no retardation. In another sense it is not uniform, for the linear velocity of the separate filaments (measured in a direction parallel to the axis of the pipe) is not the same at all parts of the cross-section.

These filament-velocities, at different parts of a circular pipe, have been measured by D'Arcy; and the general results are broadly indicated in Figs. 8 and 8*a*, which may be regarded as traced upon a longitudinal section and a cross-section of the pipe.

If the horizontal length v_o in Fig. 8 represents the observed

FIG. 8

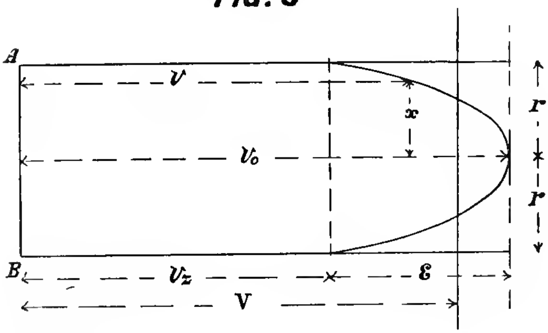
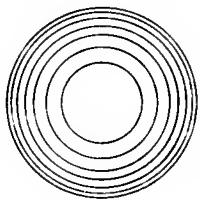


FIG. 8A



velocity at the axis of the pipe, it is found that the water creeps along the skin with a much slower velocity $v_z = v_o - e$; and at points intermediate between the centre and the circumference the velocity v is found to have intermediate values which are represented by the horizontal ordinates to the curve measured from the plane section *AB*.

The curve is, of course, the section of a surface of revolution, like the surface of a rifle-bullet; and the mean velocity V of all the filaments will be the length of a cylinder whose cubic capacity is the same as that of the bullet.

As the result of his careful measurements, D'Arcy expresses the curve by the formula—

$$\frac{v_o - v}{\sqrt{r_3}} = 11{\cdot}3\left(\frac{x}{r}\right)^{\frac{3}{2}}, \text{ in metre measure,}$$

where x is the radial distance of any filament from the axis, and r is the radius of the pipe. When $x = r$, this gives for the drag e the value $e = v_o - v_z = 11{\cdot}3\sqrt{rs}$; while the mean velocity will be—

$$V = v_o - \tfrac{2}{7}e = v_z + \tfrac{5}{7}e.$$

Turning them into English measure, these expressions may be written—

$$v_o - v = 29\sqrt{Rs}\left(\frac{x}{r}\right)^{\frac{3}{2}}$$

$$e = v_o - v_z = 29\sqrt{Rs}$$

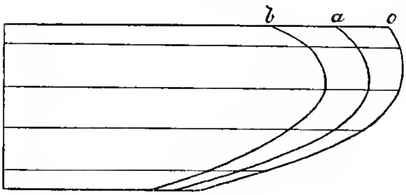
in which R is the hydraulic mean radius, equal to $\frac{r}{2}$. Or again, if c is the coefficient in the general formula $V = c\sqrt{Rs}$, we should have—

$$\frac{e}{V} = \frac{v_o - v_z}{V} = \frac{29}{c}$$

whence it would follow that the coefficient c is a measure of the ratio $\frac{V}{e}$, or is determined by that ratio.

In like manner M. Bazin has recorded a great many measurements of the filament-velocities at different points in the cross-section of open channels; and the general character of the results is indicated in the two examples sketched in Figs. 9 and 10, which refer to a channel of rectangular section having a width of about 2 metres.[1]

FIG. 9



In Fig. 9, the filament velocities are shown at three vertical sections on the lines o, a, and b, in the cross-section of Fig. 9a, the section o being taken at the centre of the channel.

The lines of equal velocity are traced upon the cross-section. In the case of the pipe these lines are all concentric circles, as shown in Fig. 8a; and here the outer ones approximate to a rectangular form, rounding off the corners of the rectangle, while the inner ones approach more nearly to an elliptical shape.

The diagrams show quite clearly that, in this channel, the filament which moves at the greatest velocity is not at the surface,

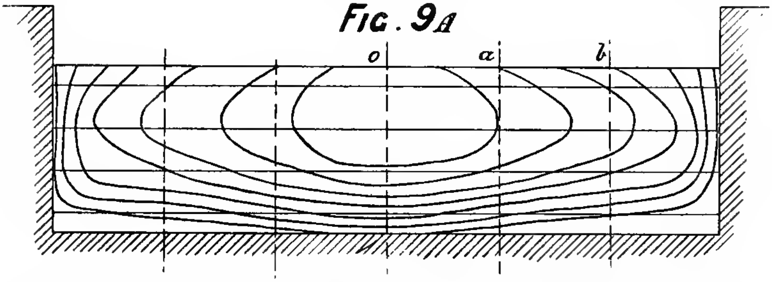
[1] "Recherches Hydrauliques." Paris, 1865.

but at some little distance below it. The velocity-curves in Fig. 9 are all bent back, and their form always seems to suggest. the action of some retarding force at the surface of the stream.

In large rivers it has sometimes been noticed that the backward curvature depends partly upon the direction of the wind. When a strong wind is blowing up-stream the curve assumes a form like that shown at *b* in Fig. 9; and it only approaches to the curve *o* when the wind is blowing strongly in the down-stream direction.

It would, no doubt, be difficult to explain these much-debated facts in language that would be universally satisfactory; but the facts offer some unmistakable indications as to the character of the work that is being done when the flow is maintained at the unchanging velocity V. Evidently the work is not wholly done in overcoming the frictional resistance (of whatever kind) between the fluid and its solid boundary, for a great deal of energy must

FIG. 9A



be expended at the same time in the mass of the fluid itself, where water is being perpetually telescoped through water.

In Art. 6, it was already remarked that, when the discharge from a pipe is delivered into the still water of a reservoir, the whole of the kinetic energy $\frac{v^2}{2g}$ is quickly dissipated by the formation of eddies; and it would be reasonable to expect that *some fraction* of the same quantity of energy is dissipated in the same way during the progress of the water along each foot of the conduit's length. For, although no part of the water in the pipe of Fig. 8 is absolutely at rest (like the water in the reservoir of Fig. 4), yet the diagram shows that there is a relative motion between the core and the outer sheaths of the cylindrical column; and this relative motion is certainly attended with the constant formation of eddies.

The sinuous path which the particles follow, and which may

be observed in a glass tube when a filament of coloured liquid is introduced at the axis, has been described by Prof. Osborne Reynolds; and the line appears to waver from one side of the tube to the other, branching off frequently into eddying whorls. We cannot suppose, therefore, that the filaments move in parallel straight lines with the velocity v, which is shown in the diagram, but which can only represent the average value of the horizontal component of their motion. A particle which flows, at one moment, along the axis, with the velocity v_0, may find itself a moment later close to the side of the pipe, where the mean linear velocity is only v_z.

Hence, it would almost appear that the work done is really of two kinds. A certain amount of energy is, no doubt, expended in overcoming the proper skin-friction through the distance v_z; and, at the same time, a further expenditure of energy must take place in the body of the fluid. It might be surmised, with some probability, that the latter will be proportional to V^2, or, perhaps, to the quantity $v_o^2 - v_z^2$; but the former may be governed by some different law.

Lastly, we may refer to the fact that, when the tube is very small, and the velocity very low, the resistance is directly proportional to V. This fact was established by the experiments of Poiseuilles upon capillary tubes, whose diameter was never greater than two-thirds of a millimetre, and was generally very much smaller. But it has been further shown, by Prof. Osborne Reynolds,[1] that the same law holds good for glass tubes up to half an inch or an inch in diameter, provided that the velocity of flow is sufficiently small. As soon as the critical velocity is reached the water breaks into sinuous eddying motion; but at lower velocities the filaments appear to move in straight parallel lines, without causing the slightest break in the central filament of coloured liquid. Hence there can be little doubt that in such examples all the filaments move with the same velocity, the whole body of fluid creeping forward like a solid; and in that case it is evident that *the whole* of the energy is expended upon skin-friction. The experiments appear to show, therefore, that in such small glass tubes the *skin-friction* is directly proportional to the velocity —so long, at least, as the velocity is very small.

[1] *Proceedings of the Royal Society*, 1883, vol. xxxv.

CHAPTER II.

EXPERIMENTS ON THE FLOW OF WATER IN PIPES.

Art. 10. Methods of Experiment.—It has already been mentioned that a very large number of observed facts must be brought together before we can venture to draw from them any conclusions upon a subject so complex as the motion of water. It may now be added that, if we try to make any use of the evidences afforded by experimental observations, a great deal will depend upon the accuracy as well as the completeness of the recorded measurements.

The quantities that have to be brought into comparison for the purposes of our inquiry are the following:—

1. The diameter of the pipe d, from which the hydraulic radius R and the sectional area a are easily found.

2. The hydraulic gradient s, or the measured loss of head h_2 upon the measured length of pipe l_2.

3. The observed discharge Q in cubic feet per second, from which the mean velocity $V = \frac{Q}{a}$ can readily be obtained if the diameter has been carefully measured.

The question will also be affected by the roughness of the pipe—a factor which can hardly be measured or specified except by a description of the character and condition of the internal surface: and moreover it will depend, to some extent, upon the temperature of the water, although this particular has not often been recorded in connection with the gaugings.

In regard to the first of these simple measurements, it is obvious that if a pipe $5\frac{7}{8}$ inches in diameter is recorded as a 6-inch pipe, it will introduce an error of 4 or 5 per cent. in the computed velocity, and an error of more than 8 per cent. in the quantity V^2 which is proportional to $\frac{1}{d^4}$. In some experiments the mean internal diameter has been carefully ascertained

by filling the pipe with water and weighing its contents; but in other cases the measurement has been less accurate, and sometimes the dimension has been taken at the nominal diameter given by the makers.

In like manner the hydraulic gradient has been determined with various degrees of accuracy. When the experiment is made in a laboratory, and upon a small scale, it is possible to bring the high-level and low-level reservoirs side by side, and to obtain a very close measurement of the total fall by means of needle-pointed hook gauges. In other cases the measurement may become a question of levelling, and the probable degree of accuracy may sometimes be judged by the quantities themselves. One would naturally expect a considerable percentage of error if the total fall were only a few inches in a distance of several miles. So also the use of any kind of piezometer, mercurial manometer, or pressure-gauge will always be attended with certain errors of observation, which may become a negligeable quantity if the length l_2 and the fall h_2 are both tolerably large, but may otherwise introduce considerable error into the results. It is unfortunate also that, in some experiments, the proper fall h_2 has not been kept distinct from the initial fall h_1 due to the first generation of velocity, and in consequence the records are sometimes ambiguous.

Perhaps, however, the greatest difficulty lies in the accurate determination of the discharge Q. In the laboratory this can be done by the use of measuring-tanks of sufficient capacity to eliminate errors of time-measurement; and when the discharge of a water-main can be turned into a closed reservoir the measurement can be very correctly made if the flow is kept uniform for a considerable time. But when these methods have not been practicable, the water has sometimes been discharged over a weir or a notch-board, and the discharge *computed* by means of a formula; while the formula employed for this purpose is itself in need of experimental verification. A closer approximation can, no doubt, be obtained by the use of a Venturi water-meter which has been carefully calibrated beforehand; but the most reliable experiments are certainly those in which the water has been delivered into some kind of measuring-tank for a given space of time, and the quantity actually measured in bulk.

For all these reasons it seems quite certain that inaccuracies are sometimes to be expected; and it is not difficult to select from

among the whole mass of gaugings some observations which are likely to be more accurate than others, because they have been obtained by more accurate methods.

Art. 11. The Inconsistencies of Experiment.—Every one who has tried to discover truth by putting an experimental question to nature, will know the pleasure that is experienced in obtaining at once what seems to be a definite answer. But if he has had the patience to verify his supposed information by further inquiry, he will certainly have found that the truth is not easily to be obtained from a passive witness who volunteers no remark upon the line of investigation which is being pursued. Very frequently the first result of such an inquiry is to elicit a series of answers which appear to be self-contradictory.

Thus, when the velocity V is repeatedly measured in the same pipe, at the same inclination s, and at the same temperature, it is certain that the same result should be repeatedly obtained; and when the results are widely different, it is impossible to accept their evidence.

In the same way it is obvious that, if the quantities V and s are related together by *any* consistent law, their relationship can be graphically represented by *some* kind of continuous curve to which the quantities V and s are co-ordinates: and if the co-ordinates as measured present a succession of zig-zagging aberrations, alternatively positive and negative, the inconsistency of such evidence suggests that the observations must be at fault in one direction or another.

Sometimes it is due, no doubt, to the use of defective apparatus; and in general it will be noticed that experiments which have been made by the most accurate methods are those which yield the most consistent results. But in other cases it has, very likely, arisen from the accidental occurrence of some unnoticed change in the conditions of the experiment.

Among the hundreds of gaugings which have been recorded in all parts of the world, it is generally to be noticed that, when the pipes have been new and clean at the time of the experiment, and protected from oxidation, the measurements are tolerably consistent; while the most incongruous results have often been obtained when the pipes have become rusty, or when they have been many years in use; and in such cases it is at least possible that the flow may have been affected by obstructions unknown to the observer.

Readers who care to examine the details of all these experiments, and the consistency of their results, may be referred to the historical review given by Mr. Hamilton Smith in his work on Hydraulics, which contains also a critical analysis of the measurements and of the various methods of experiment that have been adopted.[1]

It will not be necessary to repeat *in extenso* the conflicting results of gaugings that have been taken in old pipes, and for the present it will be sufficient to refer to the series of measurements that are brought together for purposes of comparison in Tables 4 and 7 of Art. 30. These examples will presently be referred to in detail, but it may here be remarked that they include most of the experiments which have been classed by Mr. Hamilton Smith as possessing the best claims to accuracy, while they also include a few observations of more recent date.

The slope s, in any of these examples, is evidently proportional to the work done per unit of distance, and may be regarded as measuring the resistance; while the quantity $Rs = \frac{sa}{p}$ is a measure of the same resistance per unit of the pipe's surface.

If we take the series of values given for any one of these pipes, and set off as co-ordinates the quantities s and V, or the quantities Rs and V, they will always exhibit a fairly continuous and unbroken curve, showing apparently that a definite relation exists between these quantities.

Art. 12. Surface Friction in Very Small Tubes.—The experiments of Poiseuille, which were published so long ago as 1846, have already been mentioned incidentally. The detailed measurements need not here be quoted, as they relate only to capillary tubes; but their results may be of some value as affording a very definite determination of the resistance in a case where the resistance appears to be *wholly* due to skin-friction.

Within their own limits the experiments show very clearly that the resistance varies directly, and quite exactly, with the velocity V, and also that it varies directly with a certain quantity P which may be called the specific viscosity of the fluid, and which decreases as the temperature of the water rises from 32° Fahr.

Using θ to denote the temperature in degrees Centigrade,

[1] Hamilton Smith, "Hydraulics." New York, 1886.

Poiseuille found that the viscosity at different temperatures, as compared with that at 0° Cent., may be expressed by—

$$P = \frac{1}{1 + 0{\cdot}033680\theta + 0{\cdot}000221\theta^2}$$

while his general formula gives, for the loss of head on each unit of length—

$$s = \frac{P}{k} \cdot \frac{V}{d^2}; \text{ or } Rs = \frac{P}{4k} \cdot \frac{V}{d}$$

where k is a numerical coefficient whose constant value will be 52,502 if the dimensions are taken in English feet.

It is not to be expected that these figures will have any *direct* application to the flow in large pipes or conduits; but it is generally believed that the flow is affected in *some minor degree* by the temperature of the water, and the viscosity P has sometimes been introduced as a factor in formulas which are intended for general application and which we shall presently have to consider.

Art. 13. Early Experiments in Pipes of Larger Diameter.—It is hardly necessary to refer to the details of the careful experiments of Couplet, Dubuat, and the Abbe Bossut, whose diligent inquiries after truth were begun more than a hundred years ago. During the first half of the nineteenth century they were supplemented by a few scattered gaugings which had been recorded here and there by English engineers, and also by the more systematic measurements of Weisbach. Altogether they were sufficient to show that in ordinary cases the flow is governed by a law quite different from the simple one exhibited in Poiseuille's capillary tubes, and that the resistance is more nearly proportional to V^2 than simply proportional to V. But they were not sufficiently numerous to indicate the exact relationship, nor to show how the resistance in different pipes might be affected by their diameter or by the condition of their internal surfaces.

It was not possible, therefore, that these early experiments could afford adequate ground for any other conclusion than that which is expressed by the familiar Chézy formula in which $Rs = AV^2$, or $V = c\sqrt{Rs}$. Of course the coefficient in this formula could only be a constant quantity if the following assumptions were true :—

1. That whether the pipe be large or small, rough or smooth,

and worked at high or low velocity, the resistance is always proportional to V^2.

2. That among pipes of different diameters the resistance is simply proportional to the wetted perimeter.

3. That the resistance is not appreciably affected by the roughness of the internal surface.

It is well known that every one of these assumptions has been disproved by later research, but in many instances the errors neutralize one another, so that they are not readily to be dissociated. For this reason the earliest experiments afford no conclusive evidence as to the truth or error of the second and third of these assumptions, but they were sufficient to show that the first was not quite correct. This could readily be discovered, indeed, by comparing a series of gaugings taken in one and the same pipe, for here the diameter and the roughness of the pipe would be constant quantities which could not in any way be responsible for the observed variations in the results of experiment.

Hence the attention of hydraulicians was first directed to the discovery of the true relationship between s and V, when all other conditions remain constant; and as we must discuss one question at a time, it may be well to consider here the various formulæ that have been proposed to express this relationship, and to test them by the light of recent experiment.

Art. 14. The Resistance as affected by Velocity.—Each group of observations in Table 4, and in Table 7, records a series of experiments made with the same pipe, and with different velocities of current. And in each group the individual gaugings are set down in order from the lowest to the highest velocity, the figures in column s giving the observed loss of head per unit of length, while V gives the mean velocity in feet per second. The experimental values of the coefficient c, as found in each case by the calculation $c = \dfrac{V}{\sqrt{Rs}}$, are given in the next column headed c_e; and in every group they show that the coefficient is *not* constant, but increases continuously as the velocity increases.

Thus in the series 14–23, as the velocity increases from 0·577 to 12·034 feet per second, the coefficient rises from 86·4 to 124·1. This may be an extreme example, but it illustrates a fact which is commonly to be noticed in *very smooth* pipes, and was well known to the early hydraulicians.

So far as the tables go, the evidence shows that in all these pipes the resistance is not exactly proportional to V^2 nor exactly proportional to V, but to some function of V which lies between the two, and is more like the former than the latter. As the velocity increases, the resistance increases at a faster rate than V, but at a slower rate than V^2. Such a fact, taken by itself, might perhaps be expressed by writing, in place of the old formula—

$$Rs = aV^n \quad . \qquad . \quad (9)$$

where n is an exponent greater than 1, but somewhat less than 2.

Or, again, it might possibly be expressed by writing—

$$Rs = aV^2 + \beta V \quad . \qquad (10)$$

thus making the resistance to be the sum of two quantities, of which one is proportional to V^2, and the other is simply proportional to V.

Each of these expressions has been used with several modifications, and the binomial formula (10) was proposed at a very early date. From the results of a small number of gaugings, Dubuat, Coulomb, and Prony were led to believe that the resistance consisted of two distinct parts, the skin-friction at the perimeter being simply proportional to V; and Prony's formula deduced from these gaugings was—

$$Rs = 0{\cdot}000106V^2 + 0{\cdot}0000173V \quad . \qquad (10a)$$

Eytelwein and D'Aubuisson adopted the same binomial form, introducing, however, a distinction between the two losses of head h_1 and h_2, as described already in Art. 8, and employing h_2 as the true measure of the gradient s. This correction involved only a slight change in the values of the two coefficients, a and β, without altering the general form of the expression.

It may here be remarked that the relative velocities of the different filaments, as illustrated in Fig. 8, afford some ground for the idea that the skin-friction forms only a part of the whole resistance, and that another portion of the expended energy may be dissipated in the constant formation of eddies in the internal mass, while the skin-friction may follow an independent law, and possibly may be proportional to V, as it was found to be in Poiseuille's little tubes, and as Prony supposed it to be in pipes of larger diameter.

But it will be remembered that the loss of head in Poiseuille's

capillary tubes was inversely proportional to d^2, so that Rs would be inversely proportional to d; and consequently the binomial expression might take the modified form—

$$Rs = A'V^2 + B'\frac{V}{d} \quad . \qquad (10b)$$

and this would coincide with a formula which was once proposed by Dr. Lampe.

Again, if the indications of Poiseuille's experiments are still more closely followed, and the friction treated as a quantity depending upon the temperature and viscosity of the fluid, the expression might take the form—

$$Rs = A'V^2 + \frac{P}{4k} \cdot \frac{V}{d} \quad . \quad . \quad . \qquad (10c)$$

and this modification is practically equivalent to a formula that was at one time suggested by Dr. Hagen, although the viscosity was independently determined by Hagen, and given in somewhat different terms from those quoted in Art. 12.

Before going any farther we may stop for a moment to consider how these various formulæ may be illustrated by a graphic method which will serve to determine the coefficients in each case, and to compare the results with the more extended observations of recent times.

In the old formula $Rs = AV^2$. It is evident that the corelated quantities, Rs and V, would be represented by the co-ordinates of a parabolic curve, as in Fig. 10; and if we plot the observed values of these quantities as co-ordinates, we should obtain a number of points which *ought* to lie in that parabolic curve if the formula is correct. In the same way the other formulæ might be represented by curves of a somewhat modified form.

But for our present purposes it will be much more convenient to divide both sides of the equation by V, so that the old formula becomes $\frac{Rs}{V} = AV$; and then, plotting as co-ordinates the observed values of V and of $\frac{Rs}{V}$, the points ought to lie in one straight line *OC* passing through the origin *O*, as in Fig. 10*a*. For this diagram we may choose any vertical scale that may be convenient, and on the same scale the value of the coefficient A will be measured by the inclination of the line *OC*.

If the formula were true for any one pipe, the gaugings should

yield a series of points lying in some straight line drawn through *O*. And if it were true for pipes of all diameters, the whole series of points should lie in the *same* straight line.

Taking next the binomial expression (10) and dividing both sides of the equation by V as before, the formula becomes $\frac{Rs}{V} = a V + \beta$; and the co-ordinates will be represented by a straight line, such as the line *EF* in Fig. 10*b*, in which the height

FIG 10

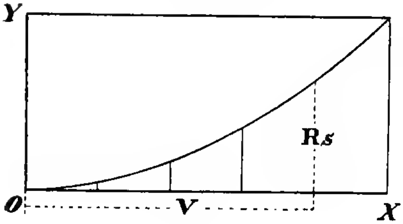
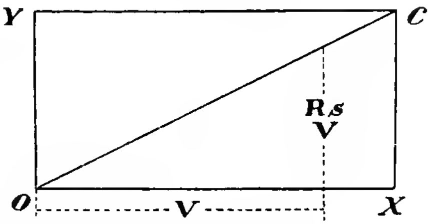


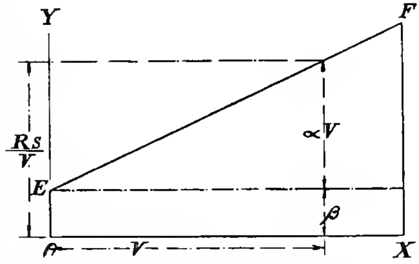
FIG 10A



OE represents the constant quantity β, while the ordinates are obtained by adding to this quantity the additional height aV proportional to the velocity.

Thus taking from any series of careful experiments the observed quantities V and $\frac{Rs}{V}$, and plotting them as co-ordinates, we should obtain a series of points which ought to lie in a straight line; and if they do so pretty nearly we can draw through them the straight line *EF*, whose inclination will give us the first coefficient a, while the height *OE* will give us at the same time the coefficient β.

FIG 10B

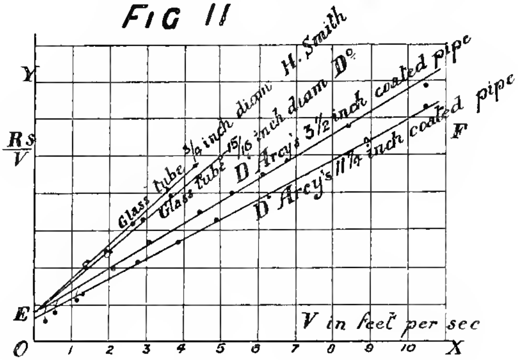


To compare the formulæ with recent observations, we may take some of the examples contained in Tables 4 and 7; and this has been done in the two diagrams Figs. 11 and 12. In Fig. 11 the line marked *EF* has beeu drawn through the points obtained from D'Arcy's series of gaugings 14–23, while the other lines in the same diagram refer to the Series 1–4, Series 5–9, and Series

24–30. Taking any one of these pipes individually, it will be seen that, for all velocities greater than about 2 feet per second, the points do indeed lie very nearly in one straight line; but it is

GROUP I — PIPES OF CLASS 1

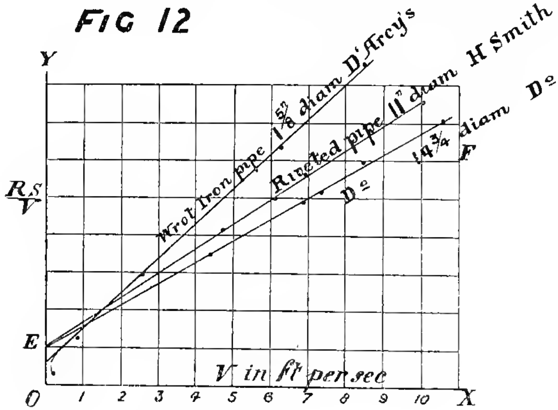
FIG 11



quite obvious that the inclination of this line, which was to determine the coefficient a, is not the same in each case, and is much steeper for small pipes than for large ones. The same thing is to

GROUP II — PIPES OF CLASS 6

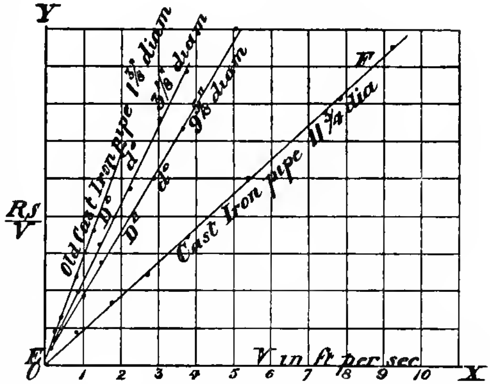
FIG 12



be seen in Fig. 12, which relates to three other series of gaugings contained in Table 7. In both diagrams it will be noticed also that the height OE is not a constant quantity.

Thence it appears that the binomial expression cannot be generally applied unless the coefficients α and β are made to depend upon the diameter of the pipe. In the modified formulæ (10_B) and (10_C) the second coefficient is represented by the quantity $\frac{B}{d}$ or $\frac{P}{4Kd}$, so that the height OE would vary inversely with the diameter; but the lines EF would all be parallel lines, while the diagrams show quite clearly that their inclination varies in a marked degree with the varying diameter.

FIG 13



In the next article we must consider how the question is really affected by the diameter, but first it may be well to note that the straight line diagram does not quite represent the relations of s and V when the current is a slow one. For velocities less than 2 feet per second, the points deviate from the straight line, and curve downwards towards the origin O. In this region it would seem that the facts are more closely rendered by the purely empirical formula adopted by Weisbach, viz.:—

$$Rs = V^2\left(\alpha + \frac{\beta}{\sqrt{V}}\right) \quad . \quad . \qquad (11)$$

which might be written—

$$\frac{Rs}{V} = \alpha V + \beta\sqrt{V} \qquad (11a)$$

Or, again, they might be represented, and perhaps still more closely, by the formula $Rs = aV^n$, referred to at the beginning of this article.

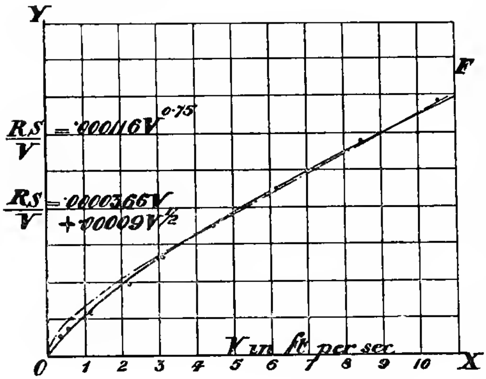
These formulæ, just like the others, will be quite inapplicable for general purposes unless the coefficients are made to depend upon the diameter; but for the sake of comparison with the straight line diagram the two curves are traced in Fig. 14 in which the results of D'Arcy's Series 14-23 are again denoted by the series of black points.

The full-line curve gives the values of $\frac{Rs}{V}$ for the equation $Rs = 0{\cdot}000116\ V^{1{\cdot}75}$, while the dotted curve is traced from Weisbach's equation, or

$$\frac{Rs}{V} = 0{\cdot}0000366V + 0{\cdot}00009V^{\frac{1}{2}}$$

in which, however, the coefficients are slightly different from those given by Weisbach. With these values for the coefficients, it will be seen that the two curves are nearly identical, and that either of them would represent very closely the results of this experiment down to the lowest observed velocities. But if the curves were transposed to Fig. 11 or to Fig. 12, they would evidently fail to show any sort of agreement with the results that have been obtained in pipes of larger or smaller diameter.

FIG 14



Art. 15. The Results of D'Arcy's Experiments.—The formulas which have been reviewed in the last article were evidently designed with the single object of expressing the relationship between the fall s and the velocity V as observed in a series of experiments made in one and the same pipe. As between pipes of different diameter the formulæ assume that the resistance will be simply proportional to the wetted perimeter, so that s would be inversely proportional to the hydraulic radius R; but the error of this assumption was clearly revealed when D'Arcy put together the results of his experiments in 1857.[1]

To the sixty or seventy gaugings which had been relied on up to that time, D'Arcy added about two hundred others, taken in twenty-two pipes of different diameters, ranging from half an inch up to 19 inches. These experiments were very carefully made,

[1] "Recherches Experimentales relatives au Mouvement de l'Eau dans les Tuyaux." Paris, 1857.

and their results brought to light several important points which had hitherto been overlooked.

(1) They showed, in the first place, that, with a given velocity V, the loss of head s is *not* directly proportional to $\frac{1}{R}$, so that, if the old formula is retained, the coefficient will not be a constant for all diameters, but must be taken at different values according to the size of the pipe.

To illustrate this particular point it will be sufficient, perhaps, to select a few of his gaugings taken in pipes of different diameters, but all worked at the same, or nearly the same velocity. Thus, when the velocity was nearly 1 foot per second, the experiments showed that the coefficient c (or K) in the formula $V = c\sqrt{Rs} = \sqrt{KRs}$ would have to be taken at the following values:—

Diameter of pipe in ft.	Coefficient c.	K
0·04	59	3481
0·087	80	6400
0·130	84	7056
0·271	99	9801
0·643	104	10816

All these, it should be remarked, were pipes of the smoothest kind, either glass or lined with some description of glazed composition; and for such pipes, D'Arcy proposed to express the average results of his experiments by using a variable coefficient $A = \frac{1}{K}$ in the old formula $V = \sqrt{\frac{Rs}{A}}$.

For the coefficient he writes $A = \alpha' + \frac{\beta'}{R}$; and if we turn the values into English measure, the formula may be written—

$$s = \frac{h_2}{l} = \frac{0{\cdot}02}{d}\left(1 + \frac{1}{12d}\right)\frac{V^2}{2g} \quad . \quad . \quad . \quad . \quad (12)$$

in which s is the loss of head per lineal foot, and d is the diameter of the pipe, in English feet.

(2) Another result of these experiments was to show how greatly the frictional resistance is affected by the smoothness or roughness of the internal surface, and thus to disprove the old fallacy about a water lubrication. The fact seems to have been

first perceived when the series of gaugings, taken in a new cast-iron pipe, were repeated a day or two later, the surface having become slightly rusted in the interval; and the discharge of the pipe was then found to be notably diminished. But the effect was still more conspicuous when a comparison was made between the observed flow in pipes with a glazed lining, and in pipes of bare metal, and, again, between the discharge of new metal pipes and of old ones, in which the surface had become sensibly eaten or pitted with rust.

D'Arcy made no attempt to classify the several degrees of roughness; but, taking the average results of his gaugings, he proposed to express the loss of head in pipes of bare metal by simply adding 50 per cent. to the value given in formula (12); and for old or very rough pipes he doubled that value, so that the formula would become—

$$\text{for bare metal, } s = \frac{0{\cdot}03}{d}\left(1 + \frac{1}{12d}\right)\frac{V^2}{2g} \qquad (12a)$$

$$\text{for old pipes, } s = \frac{0{\cdot}04}{d}\left(1 + \frac{1}{12d}\right)\frac{V^2}{2g} \quad . \quad . \quad (12b)$$

It will presently be seen, however, that the effect of roughness depends, in all probability, upon the diameter of the pipe, and is much more serious in small than in large conduits.

(3) In addition to all this, it may be observed that D'Arcy's experiments reveal another fact, which goes to modify all the earlier conclusions in regard to the relations between s and V. This matter was discussed in Art. 14, and, in accordance with the observations therein recorded, it is very clearly shown by some of D'Arcy's gaugings that, when a given pipe of the smoothest class is worked at different velocities, the coefficient rises in value as the velocity increases. But if we select from D'Arcy's experiments a series of gaugings taken in any of the older and rougher pipes, we find no such rise in the value of the coefficient. As the velocity increases, the coefficient shows very little alteration, and sometimes no alteration at all.

To illustrate this point we may just look at the values deduced for the coefficient c from two series of gaugings. In the first series, taken in a new pipe whose diameter is 0·643 feet, the velocity varies from 0·59 to 19·72 feet per second, and the coefficient rises gradually from 104·1 to 141·0. But in the second series an old pipe is employed, the diameter being 0·798 feet; and as the

velocity rises from 1·0 to 2·3 feet per second the coefficient shows a very slight increase, and no further increase at all when the velocity rises from 2·3 to 12·5 feet per second.

The recorded observations are as follows :—

Series IX. New Pipe. $d = 0{\cdot}643$.			Series XIX. Old Pipe. $d = 0{\cdot}7979$.		
V	s	c	V	s	c
0·591	0·0002	104·1	1·007	0·00094	73·6
0·912	0·00048	103·8	1·483	0·00202	73·9
1·529	0·00129	106·2	2·320	0·00473	75·5
2·559	0·0033	111·1	3·629	0·0115	75·8
3·530	0·0058	115·6	5·075	0·0229	75·1
5·436	0·0119	124·3	6·014	0·0320	75·3
5·509	0·0120	125·4	6·801	0·0410	75·2
7·411	0·0210	127·6	12·576	0·1398	75·3
9·000	0·0297	130·2			
10·013	0·0364	130·9			
19·72	0·12156	141·0			

In the case of this rough old pipe, therefore, and in many others of the same class, the loss of head is very nearly proportional to V^2; and for such pipes the old formula would not be at all improved by introducing a binomial function of V, or by expressing s as any power of V other than the square.

It was, perhaps, owing to this circumstance that D'Arcy did not propose any change in the old formula, so far as regards the relationship between s and V. And yet it is obvious from the results of Series IX., and many others, that the loss of head in a *smooth* pipe is far from being proportional to V^2. D'Arcy, however, was content to rectify the supposed relation of s to R.

Art. 16. The Resistance measured in Terms of the Velocity and of the Diameter.—We have seen that the old formula $s = A\frac{V^2}{R}$ does not express the true relationship of s to V, nor the true relationship of s to R.

The earlier hydraulicians proposed to remedy the first error by using a binomial function of V, writing $s = \frac{1}{R}(\alpha V + \beta V^2)$, and

D'Arcy proposed to remedy the second error by a similar expedient.

Thus, if we use σ to denote the reciprocal of R, or the quantity $\frac{\text{perimeter}}{\text{area}} = \frac{1}{\text{R}}$, it is evident that D'Arcy's formula may be written—

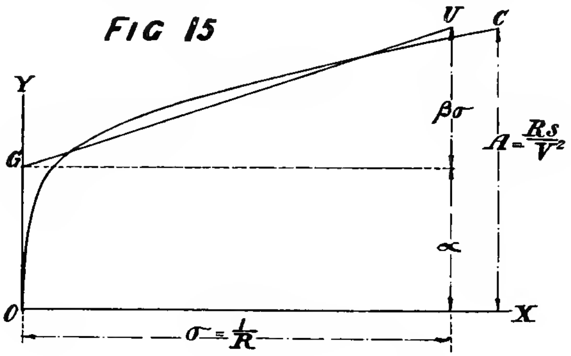
$$s = \text{V}^2(\alpha'\sigma + \beta'\sigma^2) \quad . \quad . \quad . \quad . \quad . \quad (12c)$$

and for glass tubes and glazed pipes it would become—

$$s = \text{V}^2(0{\cdot}000078\sigma + 0{\cdot}0000016\sigma^2) \qquad (12d)$$

If, for any given and unaltered value of V, this formula is taken as representing the true relations between s and R, or between s and σ, then the values of the coefficients α' and β' should

FIG 15



be determinable by the same graphic method that was employed in Art. 14 for the analogous coefficients α and β in the earlier binomial formulæ.

Dividing both sides of equation (12c) by $\text{V}^2\sigma$, we should have $\frac{\text{R}s}{\text{V}^2} = \text{A} = \alpha' + \beta'\sigma$, so that the experimental quantities σ and $\frac{\text{R}s}{\text{V}^2}$, plotted as co-ordinates, should yield a series of points lying in some straight line, such as the line GC in Fig. 15. The inclination of that line should determine the coefficient β', while the coefficient α' should be given by the height OG upon the diagram.[1]

The best experimental measurements for such a purpose would

[1] This method was proposed by Bazin for determining the constants in the case of open channels.

be a series of tests carried out in pipes of progressively increasing diameter, and all worked at the same velocity, the gradient s, which is required to maintain this velocity, being carefully measured in each case. And it would be an essential condition that all these different pipes should possess the same smoothness of surface. In the absence of such a series it is not easy to ascertain exactly how the resistance is affected by the diameter apart from other influencing conditions, nor to determine the coefficients by reference to any recorded pipe-gaugings.

The examples quoted in Table 4, along with many others that might be mentioned, agree in showing that, for any given velocity V, the coefficient A in the old formula is *not* a constant quantity for all diameters, or, in other words, that the loss of head s is *not* proportional to σ, but is rather proportional to some quantity which lies between σ and σ^2.

Here, again, therefore, it would appear that the truth, so far as it is known by experiment, might be expressed, with almost equal correctness, either by D'Arcy's binomial formula, or by writing—

$$s = \text{Constant} \times \sigma^m$$

where m is some power a little higher than unity.

If this represents fairly well the true relation between s and R at *any* given velocity, it will be possible to employ the general formula—

$$s = \mu V^n \sigma^m = \mu \frac{V^n}{R^m} \quad . \quad . \quad . \quad . \quad (13)$$

for calculating the loss of head at varying velocities and in pipes of varying diameter; and many hydraulicians are agreed in regarding this as the truest expression of all that is known upon the matter.

From a limited number of quite early experiments, made with small tubes, Dr. Hagen deduced—

$$s = \mu \frac{V^{1 \cdot 75}}{R^{1 \cdot 23}}$$

Dr. Lampe, in a similar manner, expresses the results of his large-scale experiments upon the Dantzic water-main, by writing—

$$s = \mu \frac{V^{1 \cdot 802}}{R^{1 \cdot 25}}$$

while the later researches of Prof. Osborne Reynolds have led him to

the conclusion that such an exponential formula possesses a better claim than any other to a rational foundation in theoretical hydrodynamics.

As compared with either the binominal of Prony or that of D'Arcy, taken by itself, there can be little doubt that it more correctly measures the resistance as depending upon the *two* variable quantities, velocity and radius.

So far as regards the relation between s and R, the practical difference between (12c) and (13) may be illustrated by reference to Fig. 15. It is the difference between the straight line *GV* and some curved line such as *OC*. That curve represents, in fact, the values of $\frac{\mathrm{R}s}{\mathrm{V}^2}$ for a velocity of 4 feet per second, as calculated from Dr. Lampe's values in formula (13); and it shows that at this particular velocity the two formulæ would agree pretty closely for pipes of varying sizes up to 12 inches or 18 inches diameter. But beyond this point there is a great and ever-increasing difference as R increases or σ decreases indefinitely. Here the straight line implies that the coefficient of resistance A approaches a certain minimum value *OG*, while the curve indicates that it becomes a vanishing quantity as the diameter is indefinitely increased.

Since the date of D'Arcy's experiments we have only a few gaugings in pipes of larger size. The results of Messrs. Fteley and Stearns' measurements in the 4-feet pipe of Sudbury conduit, as given in Table 4, appear to show a closer agreement with the curve of Fig. 15 than with the straight line. The observed loss of head is far less than the value that would be given by formula (12).

Art. 17. The Element of Smoothness.—For the various calculations that have to be made in engineering practice, it will be very easy to apply the general formula $s = \mu \frac{\mathrm{V}^n}{\mathrm{R}^m}$ by the aid of a table of common logarithms, as the expression may obviously be written—

$$\log. s = \log. \mu + n \log. \mathrm{V} - m \log. \mathrm{R} \quad . \quad . \quad (13a)$$

But it is quite certain that the coefficient μ and the power n will both depend upon the smoothness of the internal surface, and the power m will probably be affected by the same circumstance. It is necessary, therefore, to determine the values of these constants by actual experiment.

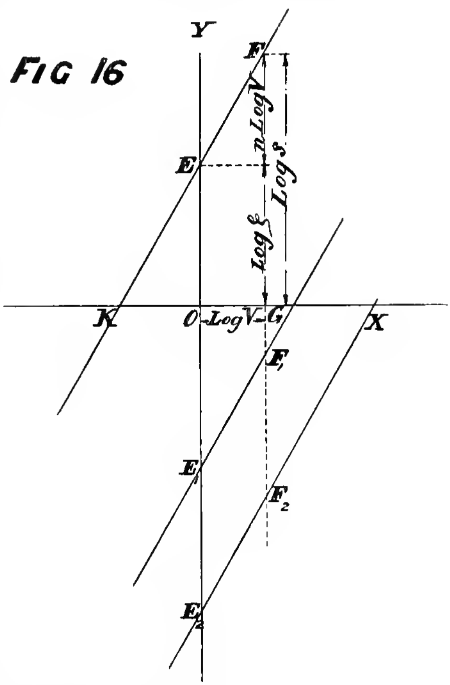
In regard to the power n it has already been mentioned that for glass tubes s is often found to be proportional to $V^{1{\cdot}75}$, while in rougher pipes it is more nearly proportional to V^2: and, in agreement with these records, Prof. Reynolds finds that n increases with the roughness of the surface. He estimates its lowest value in very smooth tubes at 1·75, and its highest value at 2.

From any series of experiments which have been made with *the same pipe*, worked at varying velocities and at different gradients, it is quite easy to find the actual value of n by the graphic method employed by Prof. Reynolds. For here the only variable quantities are s and V, while μ and R^m are constant, whatever may be the unknown value of m. Therefore, using ξ to denote $\frac{\mu}{R^m}$ we should have log. ξ = a constant, and—

$$\log. s = \log. \xi + n \log. V$$

so that if we take the observed values of s and V in such a series of gaugings, and plot their logarithms as co-ordinates, we should obtain a series of points which ought to lie in one straight inclined line, such as the line KF in Fig. 16.

FIG 16



To construct the diagram we may use the divisions upon a sheet of ordinary squared paper as a scale of logarithms.

When V is greater than 1 foot per second, log. V will be a positive quantity, and may be represented by a length such as OG set off to the right of the axis OY.

In like manner, if s were ever greater than unity its logarithm, being positive, might be represented by a height GF set off *above* the axis OX.

When the quantities are fractional their logarithms will have

a negative index, and in all such cases a negative ordinate, such as OK set off to the left of O, or OE_1 and GF_1 set off below OX, would represent a negative logarithm, *i.e.* the logarithm of $\frac{1}{\mathrm{V}}$ or $\frac{1}{s}$; but there is, of course, no difficulty in numbering the divisions of the scale so that the ordinary logarithm of the decimal fraction can be read off directly upon the scale without any conversion.

If we take from among the examples quoted in Table 4 any series of experiments made in one particular pipe, and plot as co-ordinates the logarithms of the observed V and s, we shall obtain a series of points which run very nearly in one straight line; and it will be found that the agreement is closer than in any of the straight-line diagrams previously described.

Drawing the line KEF through such a series of points, the inclination of that line gives us the experimental value of n, while the height OE measures the value of log. ξ for that particular pipe.

Repeating the same construction for another pipe of larger diameter, we should get another straight line such as E_1F_1 or E_2F_2, and if all the pipes were equally smooth we should expect all the lines to be parallel.

The experiments included in Table 4 were all made with pipes of the smoothest class, lined with some glazing composition; and in each case the line has an inclination of 1·75 to 1·00.

In Table 7 we have a few examples of pipes of bare metal, some of them with riveted joints, but all of them new and in good condition; and when these are treated in the same way it will again be found that the points run in straight lines, very nearly parallel, but having an inclination of 1·80 or 1·83 to 1·00.

The lines obtained in the same way from the gaugings of rougher pipes have generally a still greater inclination; but they are not always straight, and they are not nearly parallel. In many cases the inclination is approximately 2 to 1, and sometimes it is greater than 2 to 1.

(2) To find the true value of the exponent m in formula (13) will be a more important and also a more difficult matter. On theoretical grounds Prof. Reynolds proposes to make $m = 3 - n$, so that when $n = 2$ we should arrive at the old formula, or $s = \mu\frac{\mathrm{V}^2}{\mathrm{R}}$. It is, however, only by actual experiment that any of

these quantities can be determined with the certainty which is so desirable in engineering calculations; and to obtain an actual measurement of the exponent m and the coefficient μ, we may adopt the following graphic method.

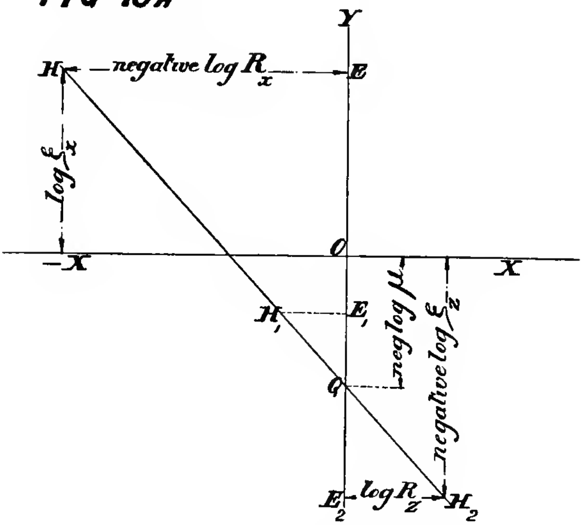
Taking a series of experiments made in pipes of different diameters, but all of them equally smooth, we may first determine by the method above described, the unknown exponent n and the quantity log. ξ for each individual pipe. The last-named quantity will vary with the varying diameters, and may be designated by ξ_1, ξ_2, ξ_3, etc., taking the pipes in the order of their increasing diameters. But in every case—

$$\log. \xi = \log. \mu - m \log. R$$

and μ will be a constant.

We can proceed, therefore, to construct another diagram, in

FIG 16A



which the co-ordinates are to be log. ξ and log. R, and we shall obtain in this way another series of points which ought to lie in one straight line sloping from left to right, such as the line HQH_2 in Fig. 16*a*.

The heights OE, OE_1, etc., representing the logarithms or the negative logarithms of ξ, ξ_1, etc., may be taken from the previous diagram (Fig. 16), while the widths E_2H_2, EH, etc., will be plotted

to represent the logarithm or the negative logarithm of R in each case. The point H will lie to the right of the axis OY when R is greater than 1, *i.e.* when the diameter of the pipe is greater than 4 feet, but for all pipes of smaller diameter it will lie to the left.

If the series of points which have been thus obtained, one for each pipe, are found to lie in any such straight line HH_2, then it is obvious that the two quantities m and μ will be measured as in all the other straight line diagrams: the inclination of the line will give us the exponent m, while the negative ordinate OQ will measure the experimental value of the negative logarithm of the coefficient μ. This method would be entirely satisfactory if the requisite conditions were fulfilled. The several experiments ought, no doubt, to be conducted at the same temperature; but the chief difficulty lies, perhaps, in the uncertainty whether the different water-mains can fairly be classed together as having the same smoothness.

Taking, in one group, the examples of the smoothest class quoted in Table 4, the graphic measurement of the three quantities n, m, and μ is shown upon one diagram in Fig. 17, and it will be seen that the second series of derived points follows pretty nearly, but not quite, exactly in the straight line HQ. The diagram gives $n = 1{\cdot}75$, $m = 1{\cdot}167$, log. $\mu = \bar{5}{\cdot}878$. or, in other words, for pipes of Class I.:—

$$s = 0{\cdot}0000755\frac{\mathrm{V}^{1\cdot75}}{\mathrm{R}^{1\cdot167}}$$

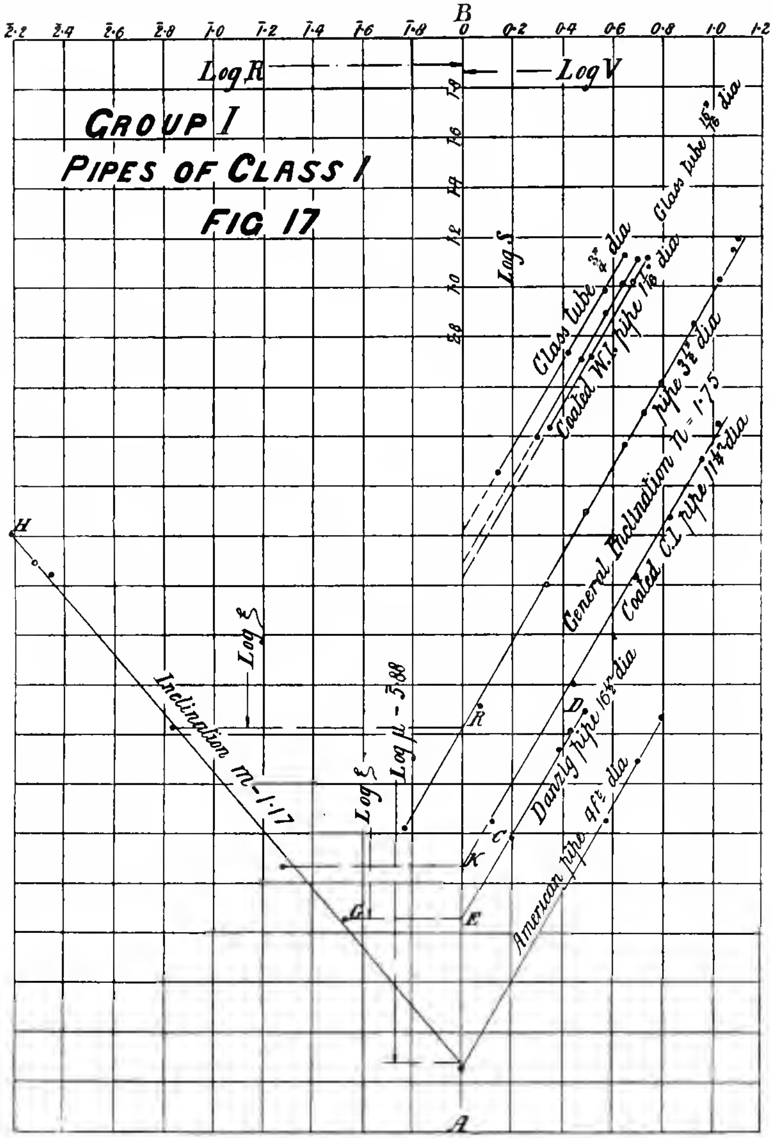
$$\log. s = \bar{5}{\cdot}878 + \tfrac{7}{4}\log. \mathrm{V} - \tfrac{7}{6}\log. \mathrm{R}$$

If we group together, in like manner, the new pipes of bare metal contained in Table 7, the results obtained from the diagram are not quite so definite. The exponent n is somewhere between 1·77 and 1·83, while m appears to lie between 1·16 and 1·20, and log μ has a value of $\bar{5}{\cdot}980$ to $\bar{5}{\cdot}920$.

In the case of rougher pipes, no definite results can be obtained. The experiments do not agree among themselves, and therefore cannot be brought under any kind of general rule; but the resistance is always high in comparison with that exhibited in the smoother pipes.

Art. 18. The Effect of Temperature.—It was found, long ago, by Dr. Hagen, that the resistance of small glass tubes depends very much upon the viscosity of the fluid, and that, in water, this

depends upon the temperature. When the temperature ranges from 32° to 212° Fahr. the coefficient μ in Hagen's formula (13)



ranges accordingly from 0·0002173 to 0·0001314; and a somewhat similar effect was observed by Prof. Unwin in measuring the

resistance of a small smooth disc rotating in the fluid. In cases like these, where the work seems to be almost wholly done against skin-friction, the resistance appears to be greatly affected by a change of temperature; but whether the same change would produce a similar or any corresponding effect upon the flow of water in a large pipe or culvert, is a question which could only be answered by actual experiment. In a few of D'Arcy's gaugings the temperature of the water has been noted; but the records do not afford any adequate material for a satisfactory answer to the question.

Prof. Osborne Reynolds, whose experiments were made with small tubes, introduces the viscosity of the fluid as a factor in the coefficient μ.

His general formula may be written—

$$s = \frac{B^n P^{2-n}}{A} \cdot \frac{V^n}{D^{3-n}} \cdot \qquad (14)$$

where D is the diameter in feet, and P the viscosity as already measured in Art. 12; while B is the constant number 36·8, and A = 1,917,000. For practical purposes it may be noted that the formula would lead to the following inferences:—

1. The effect of a change of temperature would be greatest in very smooth pipes, and would diminish with increasing roughness.

When $n = 2$, as, for example, in many rough pipes and probably in culverts of rough brickwork or masonry, the effect would disappear, for at all temperatures the quantity P^{2-n} would become unity. The resistance in such culverts would therefore be unaffected by the temperature.

2. Whatever may be the smoothness or the roughness of the pipe or conduit, the quantity P^{2-n} becomes equal to unity when the temperature is down at 32°, and at cold temperatures a little above the freezing-point, it only differs from unity by a small percentage.

Hence we may perhaps conclude that the effect of temperature may be neglected in computing the discharge of culverts or rough pipes, and is only to be considered in connection with the cold-weather or hot-weather flow in very smooth conduits.

But here, again, if the discharge falls off with a lowering of the temperature, the engineer will generally wish to calculate the flow that he can depend upon in the coldest weather; and for that purpose the factor P^{2-n} may be omitted from the calculation.

In concluding this chapter it may be remarked that pipe-experiments afford the best of all possible means for determining the relation between s and V, because those two quantities can be made to undergo simultaneous changes, without introducing any change in R or in the element of smoothness.

But they do not afford the best means of studying the relation between s and R, or of fixing the value of the exponent m, because, in passing from one pipe to another of larger diameter, we can hardly be sure that we are conducting the second experiment upon a surface of the same smoothness. For this purpose we should rather have recourse to experiments in open channels, for here it will be possible to measure the varying resistance which is exhibited by one and the same channel with varying values of the hydraulic radius R.

CHAPTER III.

EXPERIMENTS ON THE FLOW IN OPEN CHANNELS.

Art. 19. Early Observations.—Until the publication of Bazin's "Recherches Hydrauliques" in 1865, very little information had been obtained as to the conditions which really determine the resistance of an open conduit. Most of the early writings have reference to the flow of natural streams or the discharge of canals executed in earthwork; and everybody seems to have accepted the old formula $s = A\frac{V^2}{R} = \frac{V^2}{KR}$ as the best of all possible expressions.

Thus Eytelwein, Beardmore, Neville, Leslie, and D'Aubuisson calculate the resistance in terms which are practically equivalent to the old formula, but choosing independently for the coefficient A certain arbitrary values which differ slightly from each other.

For example, Beardmore finds—

$$V = 94{\cdot}2\sqrt{Rs}; \text{ or } s = \frac{V^2}{8874R} \quad . \qquad (6\text{A})$$

while D'Aubuisson makes $c = 95{\cdot}6$, or $K = 9140$; applying these values, however, to currents whose velocity is not less than 2 feet per second.

In the same way, Neville also finds it necessary to distinguish between very slow currents and currents running at a higher velocity, choosing for K the value 8519 in the one case and 8705 in the other.

On the other hand, Leslie found that the coefficient must be chosen with some regard to the hydraulic radius. For small streams he proposed to take $c = 68$, or $K = 4624$, while in the case of larger rivers he would increase the value to $c = 100$, or $K = 10{,}000$. Already these hydraulicians had discovered that s is not proportional to V^2, nor inversely proportional to R, and that the coefficient must therefore be taken at varying values which

would depend upon V and R; but they had greatly underestimated the variations that would have to be applied.

The later gaugings of natural water-courses have shown variations far greater than anything they had anticipated; and even if we exclude the streams that flow in irregular beds, and confine ourselves to the discharge of artificial conduits of moderate dimensions constructed in regular prismatic forms, we yet find that the coefficient will sometimes be as low as $c = 30$ or $K = 900$, while in other cases it will reach the value $c = 150$, or $K = 22{,}500$.

These observed variations are so wide that they almost seem to render the old formula useless. The causes to which they are due have, however, been pretty clearly elucidated by the experiments of M. Bazin, which must next be referred to.

Art. 20. Bazin's Experiments.—Following up the work of D'Arcy, M. Bazin carried out a prolonged series of observations in open channels, and they were conducted in such a systematic manner as to show how the velocity of the current is affected by each one of three varying conditions—the roughness, the radius, and the slope of the channel.

The greater part of the experiments were conducted in a channel especially constructed for the purpose, and lined sometimes with one material and sometimes with another, taking the ordinary materials of construction from the smoothest to the roughest. The channel was laid at three different gradients, and the supply was controlled by specially designed sluices. The whole length of this experimental canal was nearly 2000 feet, while its width was 6·6 feet, and its maximum depth about 3·1 feet; the actual depth of the stream in the several experiments being regulated at pleasure.

Lined with a given material, and worked at a given gradient, a series of gaugings is obtained giving the mutually dependent values of V and R, while no disturbing change is made in the slope s or in the roughness—information which could hardly be gathered from any pipe-experiments. Lined with another material, and worked at the same gradient, another series of gaugings is obtained which can be compared with the first; and thus the specific hydraulic resistance of different materials of construction can be experimentally determined.

Finally when these successive groups of series of experiments have been repeated two or three times, and each time upon a

different gradient, it becomes possible to trace the separate effect of slope.

By such means M. Bazin has shown how the coefficient A in the old formula is affected by differences of roughness, of radius, and of slope; and, to pursue the question still further, he has varied the cross-section of the channel, conducting several groups and series of experiments in canals which were successively made in semicircular, rectangular, triangular, and polygonal forms.

As an example of thorough experimental research, carried out by one observer for the benefit of a generation of engineers, these hydraulic experiments will rank with the classical steam-measurements of Regnault.

The gaugings furnished by the experimental channel were supplemented by others which were obtained from conduits already existing, and together they serve to determine the resistance for surfaces of varying degrees of roughness, which may be classified in two groups as follows:—

(*a*) *For Masonry Conduits.*

1. A very smooth surface of neat Portland cement.
2. A smooth rendering with 2 of Portland cement to 1 of fine sand.
3. Ashlar masonry.
4. Ordinary brickwork.
5. Damaged ashlar with some weedy growth.
6. Hammer-dressed masonry.
7. Dry rubble.
8. Coarse rubble with rocky bottom.

Among these different classes of material the resistance shows wide variations, its several values standing in the order in which the materials are here placed. The coefficient A, which M. Bazin determines experimentally for each material, increases regularly as we pass from the smoothest to the roughest of the surfaces, its value for the coarse rubble being eight or nine times as great as for the smooth lining of neat cement. And we may take this coefficient as a direct measure of the observed resistance, for in each case it expresses simply the observed loss of head s per unit of length for a current of unit velocity in a channel of unit radius.

These general results are confirmed by comparing the values obtained from the second group of materials, viz.:—

(*b*) *For Channels in Timberwork and Earthwork.*

1. Carefully planed board.
2. Sawn planks, not planed.
3. The same roughened by transverse strips 1 inch × $\frac{3}{8}$ inch, nailed down the walls and across the bottom with intervals $\frac{3}{8}$ inch wide.
4. Channel lined with small pebbles $\frac{3}{8}$ to $\frac{7}{8}$ inch wide.
5. Channel lined with coarser pebbles $1\frac{1}{4}$ to $1\frac{1}{2}$ inch.
6. Channel as in No. 3, the transverse strips being placed at intervals 2 inches wide.

Here, again, the resistance is found to increase with the roughness all through the list. For the smoothly planed board the coefficient appears to have nearly the same value as for the smooth rendering of neat cement, or for the glazed lining of the smoothest pipes; and from this minimum value it rises continuously through a range which is nearly as great in group (*b*) as it is in group (*a*).[1]

The experiments show in a very forcible manner how important is this question of roughness.[1] At the same time they show that, whatever may be the character of the lining, the resistance in any given channel is not inversely proportional to R; for in every case the coefficient A becomes smaller as the hydraulic mean depth is increased. This is everywhere to be seen in the several series of gaugings which are reproduced in Tables 5 to 11, pp. 78–87; but before proceeding to consider the figures in detail, it may be well to refer to the information which has been obtained by other observers.

Art. 21. Later Experiments.—If we have to calculate the discharge of a large channel by reference to experiments made in conduits of smaller dimensions, it will be highly important to determine the true relationship between *s* and R; and what is needed, more than anything else for this purpose, is a series of large-scale experiments. The hydraulic radius R, in Bazin's channel, was generally less than 1 foot; but the careful gaugings of Messrs. Fteley and Stearns in the Sudbury conduit have extended

[1] The very highest resistances appear to be experienced when the water flows over or through the interstices of a *fibrous* material, such as the weedy growth which sometimes covers the bottom of a watercourse, or the canvas lining which was attached to the wooden channel in one of Bazin's small scale experiments; and this may account for the excessive values obtained by some gaugings in small rivers.

over a wider range. The culvert is 9 feet in diameter, and covered with a semicircular arch, while the floor is a dished invert, and the side-walls have a curved batter.

The lining is of hard, smooth brick, with carefully pointed joints, and the deduced values of A indicate that the "roughness" of this surface (hydraulically measured) is nearly equal to that of Bazin's cement rendering (No. 2), but is slightly less than that of his channel of ashlar masonry, and decidedly less than that of his rectangular channel of ordinary brickwork.[1]

The gaugings that were taken on a length of 4200 feet in this culvert are quoted in Table 6, p. 80, from Hamilton Smith's "Hydraulics."

For the first series the current was worked at a nearly uniform gradient and uniform depth; but the gradient of the water-surface was adjusted for the remaining series by damming the water at one end or the other. When the culvert was worked in this way the cross-section of the stream was, of course, not uniform throughout, so that there would be a positive or negative loss of head due to acceleration or retardation of the flow. It would be difficult, perhaps, to calculate accurately the allowance that should be made for this; but it amounts to very little on so great a length, and, as an approximation, the velocities V_1 and V_2 at the two ends have been calculated, and the quantity $\frac{V_2^2 - V_1^2}{2g}$ has been added to, or subtracted from, the total fall recorded by the observers.

We have presently to consider the conclusions that may be drawn from the valuable observations which have hitherto been mentioned. In addition, we have also the earlier experiments of Rittinger in Hungary, the later measurements of Kutter in Switzerland, Major Cunningham's gaugings in Indian aqueducts, and Mr. Gordon's on the Irawadi. Hydraulic engineering owes a permanent debt of gratitude to the men who have thus laboured for the discovery of truth, and have recorded for our information the results they have obtained. And, besides these, we have a great many isolated records evincing the same patient accuracy of observation; but we need not refer to them in detail, because, being isolated, they will scarcely enable us to trace the relations between the several variable quantities.

[1] In some measure this difference between the two brickwork channels may be due to the different forms of their cross-section, as will presently be noticed; but something is probably due to the character of the brickwork.

Art. 22. The Effect of Roughness.—It is not easy, even with the aid of such systematic experiments, to discover how the resistance of a channel is affected by each one of the varying conditions—velocity, radius, and roughness; but if we take first the element of roughness, we can get a broad view of its effects by comparing together the several values of the coefficient A which have been observed in different channels running at the same hydraulic mean depth R. These can be found by interpolation from the tables, with sufficient accuracy for our present purpose.

Thus, putting all the experiments in one list, and taking a depth of 0·6 feet, we should find a series of values for K or $\frac{1}{A}$ such as the following:—

	Form of Channel and Character of Lining.	Coefficient $K = \frac{1}{A}$
1.	Semicircular, neat cement	19000
2.	Rectangular, ,, ...	18800
3.	Semicircular, cement and sand	15400
4.	Sudbury culvert, hard smooth brick ...	14400
5.	Various forms in sawn timber, at various slopes	from 14400 to 11500
6.	Ashlar masonry	12200
7.	Rectangular, ordinary brickwork	11600
8.	Timber channel with cross strips at spaces of $\frac{3}{8}$ inch	8100
9.	Channel lined with small pebbles	6400
10.	Damaged ashlar with weedy bottom ...	6100
11.	Hammer-dressed masonry	5400
12.	Dry rubble	5300
13.	Channel lined with coarse pebbles, $1\frac{1}{2}''$ ...	3800
14.	Timber channel with cross strips at spaces of 2 inches	3600
15.	Coarse rubble with rocky bottom	2100

For example, if we were using these values in the old formula for the purpose of calculating the requisite fall of a channel, we should have to write for the channel of neat cement—

$$s = \tfrac{1}{19000} \times \frac{V^2}{R}$$

and for the coarse rubble $s = \tfrac{1}{2100} \times \dfrac{V^2}{R}$

the gradient required in the latter case being nine times as great as in the former, if the radius R is about 0·6 feet in both cases, and if the velocity V is to be the same in both channels.

But while these figures may serve to illustrate the order in which the different materials stand in the scale of hydraulic resistance, we must not assume that in all cases they will represent numerically the relative values of the resistance, or that *any* constant ratio subsists between the frictional resistance of one material and of another, if that resistance is to be measured by the loss of head. They may be taken as measuring that ratio pretty nearly for channels of one particular size, but not for channels of all sizes; and this will become evident upon closer examination of the experiments.

Thus, if we take another hydraulic depth smaller or greater than 0·6 feet, and interpolate a new series of values for A or K, we shall find in either case that the several materials stand in the same order as before, but the ratio between the several values will not be the same; for the effect of roughness is most conspicuous when the depth is very shallow, and becomes less conspicuous as the hydraulic mean depth is increased. To illustrate this point we may conveniently place side by side the results obtained in a pair of channels lined with materials of different and well-defined degrees of roughness, such as No. 9 and No. 13 in the list above given—the channel being lined in the first case with pebbles $\frac{3}{8}$ to $\frac{7}{8}$ inch in diameter, and in the second case with coarser pebbles, $1\frac{1}{4}$ to $1\frac{1}{2}$ inches: while the cross-section and the inclination of the channel were the same in both trials.

Taking the actual values of A_9 as observed in the first, and interpolating the corresponding values of A_{13} in the second trial, we may obtain the following series at different hydraulic depths R:—

	Lining of— Small Pebbles	Lining of— Coarser Pebbles	
Radius R	A_9	A_{13}	Ratio $\frac{A_{13}}{A_9}$
0·357	0·000201	0·000385	1·92
0·520	0·000175	0·000293	1·69
0·644	0·000161	0·000256	1·59
0·785	0·000147	0·000231	1·58
0·871	0·000145	0·000216	1·49
0·910	0·000144	0·000210	1·45

In all probability the ratio would come nearer and nearer to unity if the depth were still further increased.

A similar comparison may be made between the two channels No. 8 and No. 14, or between No. 2 and either of the rougher rectangular channels, and in every case with similar results.

The effect of a *change* of roughness, in substituting one material for another, is very great when the water flows in a thin sheet over the bottom, and is much smaller when the stream attains a greater depth.

Hence the very great disparity which is found to exist between the resistance of different materials in Bazin's experimental canal, from smooth cement to coarse rubble, is not likely to be found in conduits of larger dimensions. As the hydraulic depth is increased the disparity between all these materials will become less conspicuous. This is really shown by the experiments themselves up to a certain range, and it is nothing more than might reasonably be expected. For it is impossible to suppose that the flow of a great river, 50 or 60 feet deep, would be sensibly retarded if its bed were strewn with 1-inch pebbles instead of half-inch pebbles; although it has just been shown that when the stream is about 4 inches deep the resistance would be nearly doubled by such a change in the character of the bed.[1]

On this particular point M. Bazin's gaugings have afforded clearer evidence than any that could be gained by means of pipe-experiments.

Art. 23. Form of Cross-section.—The comparative experiments of M. Bazin have shown that the coefficient A is not very much affected by the form of cross-section when other things are equal.

Taking a pair of timber channels laid at the same inclination $s = 0{\cdot}0015$, we find that in the rectangular form, when $R = 0{\cdot}912$ the coefficient c is $119{\cdot}1$; and when $R = 0{\cdot}911$ in a channel of octagonal form, c is found to be $118{\cdot}9$.

Comparing the rectangular wooden channel with another of trapezoidal form, both of them laid this time at a gradient $s = 0{\cdot}0049$, we find the following values:—

	R	c
Rectangular form	0·698	117·9
Trapezoidal form	0·707	119·8

[1] It may be noted here that this relation between the depth and the effect of roughness is not expressed by formula (14) of Art. 18. The formula, indeed, would seem to imply a relation of an opposite character if n increases with the roughness up to a maximum value of 2.

And, again, when the wooden trough is made in the form of a right-angled triangle, with sides inclined at 45°, and laid at the same inclination of 0·0049, the coefficient is found to be 118·4 when $R = 0{\cdot}719$.

That is to say, in each of these different forms, the resistance per unit of wetted perimeter is very nearly the same.

The coincidence is not quite so close when the channels are worked at a shallower depth of water; but for all practical purposes the differences are too small to be worth any further consideration.

At each corner of a rectangular trough, however, the flow is undoubtedly impeded by the combined resistance of the floor and the side wall; and in these corners there is an area of comparatively dead water, which is absent in the circular or semicircular section. We find, accordingly, that when the semicircular channel is running full, the resistance is perceptibly smaller, and therefore the coefficient K or c considerably greater, than in any of the sections hitherto mentioned.

Thus the wooden semicircular channel, 4 feet 7 inches in diameter, when flowing with a depth of 1·53 to 1·68 feet, so that R was between 0·856 and 0·921, seems to have yielded a coefficient $c = 125$ nearly; and this was obtained with a gradient $s = 0{\cdot}0015$, the same as in the first pair of trials above mentioned. But we have seen that the rectangular channel, laid at the same slope, gave $c = 119{\cdot}1$ when R was 0·912.

Here, we appear to have a difference in favour of the semicircular form amounting to about 5 per cent. in the coefficient c, which would be equivalent to about 10 per cent. in the coefficient A or K.

A slight difference in the same direction will be found, again, if we compare the cement-lined channel of rectangular form with the open semicircular channel, 4 feet 1 inch in diameter, lined with the same smooth rendering of neat cement: but, for the purpose of this comparison, it is unfortunate that the two trials were not made at the same gradient.

Lastly, if we compare the gaugings of Mr. Stearns in the 4-foot Rosemary pipe, with those taken by M. Bazin in the cement channel of slightly greater diameter, we find the resistance of the channel to be no greater, but rather less than that of the pipe. In the pipe we have $R = 1$, and at gradients of 0·00122 and 0·00185 the coefficients were $c = 142{\cdot}1$ and $c = 144{\cdot}1$; while

in the open channel, upon a gradient of 0·0015, we find $c = 153$ when $R = 0·992$. The smoothness of the cement channel may, perhaps, be a little superior to that of the pipe including its numerous joints.

Art. 24. The Resistance as depending upon the Hydraulic Mean Depth.—The channel experiments have yielded a great deal of information which could not be obtained from pipes. They have not only extended our measurements of resistance over a wide list of materials, but they also furnish for each separate material the best possible means of tracing the relation between s and R, because in the open channel there is no difficulty in obtaining a series of gaugings at different values of the hydraulic depth R, while the water continues to flow over the same lining.

Thus, if we had a long wooden trough which was capable of being easily adjusted at different gradients, we should find that, with a certain gradient s_1, and a certain hydraulic depth R_1, the current would flow at a certain observed velocity V_1. With another hydraulic depth R_2 greater than the first, we could certainly obtain the same velocity by adjusting the trough to a lower inclination s_2, and so on. In such a series of experiments, s and R would be the only varying conditions, and the relation between them could be directly ascertained.

This is not exactly the way in which the gaugings have been taken, and we shall presently have to consider how the same information is to be deduced from the results that have been actually recorded, for in those observations the velocity V has not been constant through any series of trials.

Without any such analysis, however, it is pretty clearly to be seen from the tables that the resistance is very far from being proportional to $\frac{1}{R}$, and the disparity is more conspicuous than it was in the pipe-experiments. The coefficient A or K deduced for the old formula does not, therefore, remain constant through a series of gaugings taken in the same aqueduct.

In the tables the gaugings are arranged in the order of the successive values of the hydraulic depth R; and as R increases the coefficient K increases also. It is important to notice that this is true of *every* series of gaugings, and for *all* materials, from the smoothest to the roughest. We might express the fact by saying that, for all materials, s is proportional to some quantity

which lies between $\frac{1}{R}$ and $\frac{1}{R^2}$, or that it is proportional to some power of $\frac{1}{R}$ greater than unity. A mathematical expression might be found for this fact, either by using a binomial of $\frac{1}{R}$ to measure the coefficient A, or by writing $s = \frac{1}{R^m} \times$ a constant, the power m being something greater than unity in *all* cases; its value will depend upon the roughness, and the facts mentioned in Art. 22 go to show that m must *increase* as the roughness increases, and cannot decrease, as is implied in formula (14).

Art. 25. M. Bazin's Formula.—Keeping to the same general lines that had been followed by D'Arcy, Bazin was content to work with the old Chezy formula $s = A \cdot \frac{V^2}{R}$, making the coefficient A an adjustable quantity depending upon R, and calculating its value by the secondary formula $A = \alpha' + \frac{\beta'}{R}$.

But his extensive research enabled him to go farther than his predecessor, and to determine for each material of construction the experimental value of the constants α and β.[1]

For this purpose he devised the graphic method, which has already been referred to in Art. 16, and illustrated in Fig. 15.

Taking a series of gaugings, which have been obtained in the same channel, worked at the same gradient, the observed values of $\frac{Rs}{V^2}$ are plotted as ordinates to the corresponding abscissæ which represent progressive values of σ or $\frac{1}{R}$. The points thus fixed are found to lie nearly along a straight line *GU* (Fig. 15), whose inclination determines the quantity β', while the height *OG* measures the quantity α'.

Classifying all the materials of construction under four heads, and taking an average for the several materials grouped together in each class, M. Bazin finds in this way the values of α' and β', whose equivalents in English measure are given in the following table:—

[1] *Vide* "Recherches Hydrauliques." Paris, 1865.

TABLE 2.—CONSTANTS FOR THE FORMULA OF D'ARCY AND BAZIN.

Class.	Materials.	α'	β'
I.	Smooth cement, Carefully planed wood	0·000046	0·0000045
II.	Ashlar masonry, Brick, Unplaned wood ...	0·000058	0·0000133
III.	Rubble masonry ...	0·000073	0·0000600
IV.	Earth	0·000085	0·0003500

The results that are broadly expressed by these figures constitute a very great advance in our knowledge of hydro-kinetics.

1. They consistently show the effect of roughness, and also its relative effects in culverts of different sizes.

When R is very great, the term $\frac{\beta}{R}$ becomes indefinitely small, and the coefficient approaches to the limiting value $A = \alpha$; and for a velocity of 1 foot per second $s = A$.

2. For any given velocity V, the formula expresses the loss of head s as a binomial function of $\frac{1}{R}$; and it may be written—

$$\frac{s}{V^2} = \alpha\left(\frac{1}{R}\right) + \beta\left(\frac{1}{R}\right)^2$$

Within the somewhat limited range that was afforded by the experimental channel, the observed facts are closely interpreted by this expression—as closely, indeed, as they could be by any other; but it will always be a difficult matter to deduce from small-scale experiments any rule that can be confidently applied to very large watercourses. If we elect to trace the relationship by writing, with Hagen,—

$$\frac{s}{V^n} = \mu \cdot \left(\frac{1}{R}\right)^m$$

making m somewhat greater than unity, we shall get, for any given velocity V, varying values of the coefficient A such as would be represented by the curve OC in Fig. 15, while the straight line GV would represent the values yielded by the

binomial. For small hydraulic depths up to about 1 foot, the two lines might run very close together, but the difference would begin to show itself as soon as $\frac{1}{R}$ is much less than unity: and for very large values of R the curve falls towards the origin *O* and lies far below the straight line.

The sparse indications that are to be gathered from a few experiments in larger aqueducts are in favour of the curve; while the gaugings of large American rivers have often yielded a coefficient A very much *smaller* than the minimum limiting value a' as given in the above table.[1]

3. For any given hydraulic radius R, the binomial formula assumes that s is proportional to V^2, and proposes no departure in this respect from the old rule. The assumption appears indeed to be very nearly correct in the case of rough masonry culverts; but if it should prove to be inaccurate when applied to smoother materials, the quantity $\frac{s}{V^2}$ in the above formula would obviously change its value with any change in the gradient s, and this would introduce a disturbing element in the calculation.

The experiments show this change in the case of the timber channel when the gradient is changed from 0·002 to 0·0049 and again to 0·008; and the evidence is repeated in several other groups of successive trials, the change being most conspicuous in the case of a small trough of smoothly planed wood. For such very smooth surfaces it would seem that, in channels as in pipes, s varies with $V^{1 \cdot 75}$. It should not be impossible, however, to eliminate the disturbing element, and the problem must be considered in the next chapter.

[1] By a modification of the formula recently suggested by M. Bazin, the limiting value of A is brought to the same figure for different materials, and the line approaches more nearly to the curve.

CHAPTER IV.

DETERMINATION OF CONSTANTS IN A GENERAL FORMULA.

Art. 26. The Logarithmic Formula.—The pipe-experiments discussed in Chapter II. have shown that, when R is constant, the loss of head at different velocities can be expressed by making s proportional to V^n, but they do not show us how s varies with R.

The channel-experiments described in Chapter III. have shown that, when V is constant, the loss of head varies with $\frac{1}{R}$ and may probably be expressed by making s proportional to $\left(\frac{1}{R}\right)^m$.

Writing, therefore, with Hagen, Lampe, and Prof. Osborne Reynolds, $s = \mu \frac{V^n}{R^m}$, the formula will express these relations in all cases; but the quantities μ, n, and m will depend in each case upon the roughness of the material, and must be experimentally determined for each material. In this chapter it will be our task to find those values.

Obviously the formula would have no meaning at all unless it can be shown that the quantities μ, n, and m have a definite value for each material, and a value which remains sensibly constant under all variations of V and of R within the limits of known experiment.

Of the two exponents n and m, the first was very well determined by the pipe-experiments; and in the smoothest glass or glazed pipes it is certain that n is very nearly 1·75, increasing to perhaps 2·0 or something more as the roughness increases; and the same values should apply to similar surfaces in open channels.

To determine the exponent m we must look chiefly to the channel-experiments, in which the measurements have been repeated for different values of R over the same lining; but the

exponent cannot be *directly* obtained from Bazin's experiments for the reason above mentioned. The determination would be easy if we might assume that s is proportional to V^2, or to any constant power of V; but it is quite certainly to be expected that, in channels as in pipes, the exponent n will vary with the roughness increasing from a minimum value of 1·75 to a much higher value in rough channels.

Art. 27. Analysis of Channel-Experiments.—We have a large number of these experiments in which a series of gaugings, or more than one series, have been carried out in *the same channel;* and throughout such a series we have said that the values of μ, n, and m must remain constant. Hence the ratio of n to m must also remain constant; and if the ratio $\phi = \frac{n}{m}$ is first determined, as it may be, from the experiments on any given channel, it will then be possible to fix the values of n and m for that material.

The formula $\log. s = \log. \mu + n \log. V - m \log. R$ may obviously be written $\log. s - \log. \mu = m\{\phi \log. V - \log. R\}$; and for all gaugings taken at the same gradient s_1, we certainly have $\log s_1 - \log. \mu$ a constant quantity. Or if we use ψ_1 to denote $\frac{\log. s_1 - \log. \mu}{m}$, we shall have $\psi_1 = \phi \log. V - \log. R$; and ψ_1 also will be constant.

Taking, therefore, any such list of successive observations in a channel or channels of one and the same material laid at one gradient s, and plotting as co-ordinates the logarithm of R and the corresponding logarithm of V, we shall obtain a series of points which ought to lie in one straight line, such as the line BP_1 in Fig. 18; and the inclination of that line will measure the value of ϕ or the ratio $\frac{n}{m}$ for that particular material.

In Fig. 18 ordinates measured to the right of O, and above O, represent the positive logarithms of V and R when those quantities are greater than unity, so that the co-ordinates of the point B, for example, are $OA = \log. R_b$, and $AB = \log. V_b$. Lower values of the depth R will give currents of lower velocity V, and when their logarithms are plotted as co-ordinates we will suppose the successive points to lie along the line BQ_1P_1. For the point Q_1, $\log. V = OQ_1$, while $\log. R = O$, or $R = 1$ foot. For the point P_1, $\log. V = O$ or $V = 1$ foot per second, while OP_1 represents the negative

logarithm of the fractional hydraulic depth R_p at which that velocity is observed.

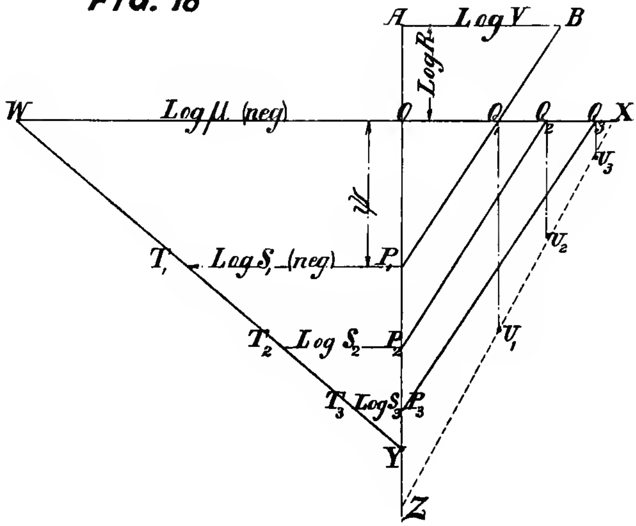
The inclination of the line is measured by the ratio $\frac{OP_1}{OQ_1}$ $= \phi = \frac{n}{m}$; and the height OP_1, which is equal to $AP_1 - AO$, or ϕ log. V − log. R, represents the constant quantity ψ_1. From the construction of the diagram it will be seen that the line P_1Q_1B at every point fulfils the equation—

$$\log. V = \frac{1}{\phi}(\psi_1 + \log. R)$$

$$\text{for example } AB = \frac{1}{\phi}(OP_1 + OA)$$

Having thus determined the ratio ϕ for the given material, and having found the value of the quantity ψ_1 when the channel is

FIG. 18



worked at the gradient s_1, we can turn to another series of gaugings which have been taken in the same channel when laid at a different gradient s_2, and treat them in the same manner.

The logarithms of the corelated quantities V and R in this second series will give us another row of points lying in some such

line as Q_2P_2, and if a third set of gaugings has been taken at a third gradient s_3, we may in like manner obtain a third row of points on such a line as Q_3P_3.

Each series would give an independent determination of the ratio $\phi = \frac{n}{m}$; but as the material has not been changed, the lines should all be parallel.

It is evident, however, that each set of observations determines a new value for the height $OP = \psi = \frac{1}{m}(\log. s - \log. \mu)$; for this quantity must change with every change of the gradient.

For three or more gradients s_1, s_2, s_3, etc., we may use ψ_1, ψ_2, ψ_3, etc., to denote the heights OP as found by the graphic construction, and these ascertained values will enable us at once to determine the constants μ, n, and m for the given material.

For if μ is constant, we have in each of the three cases the rectilinear function—

$$m\psi = \log. s - \log. \mu.$$

Therefore, if we plot once more the quantities ψ_1, ψ_2, ψ_3, as co-ordinates to the quantities $\log. s_1$, $\log. s_2$, $\log. s_3$, we should obtain three points such as T_1, T_2, T_3 in Fig. 18, and if the theory holds good these points should lie in one straight line such as the line WY.

The inclination of that line with the vertical or the ratio $\frac{OW}{OY}$ will now give us the value of m, while the length OW will represent the negative logarithm of the coefficient μ.[1]

When m has thus been experimentally measured it will be easy to calculate $n = \phi m$, or, if we like, we can measure it upon the diagram. For it is evident that in each trial $OQ = \frac{1}{\phi} \cdot OP$, and if the ordinates Q_1V_1, Q_2V_2, etc., are set off vertically to represent $\log s_1$, $\log s_2$, etc., the points V_1, V_2, V_3 will lie in a straight line ZX, whose inclination with the horizontal will give the constant n: *i.e.* $n = \frac{OZ}{OX}$; while the construction must also give $OZ = OW = \log. \mu$.

[1] To save space it may be more convenient to plot the ordinates P_1T_1, P_2T_2, etc., to represent $\log. 1000\, s_1 = 3 + \log. s_1$, and so on. The length OW will then give $\log. 1000\, \mu$, or $3 + \log. \mu$.

For practical purposes either half of the diagram is sufficient.

Art. 28. Results obtained from the Diagram.—The method will be fairly illustrated if we take first the three sets of gaugings recorded by M. Bazin for a rectangular channel of unplaned timber, laid at three successive gradients, 0·00208, 0·0049, and 0·00824. The details are given in Table Series 106–117, 118–129, and 130–141; and the logarithms of V and R have been plotted as co-ordinates in Fig. 19, giving 12 points for each series. The points lie very nearly in three straight lines, except the lowest points, and, as these represent gaugings taken in extremely shallow water, they may almost be left out of account. The three lines are quite parallel, and their inclination fixes the value of ϕ at 1·50.

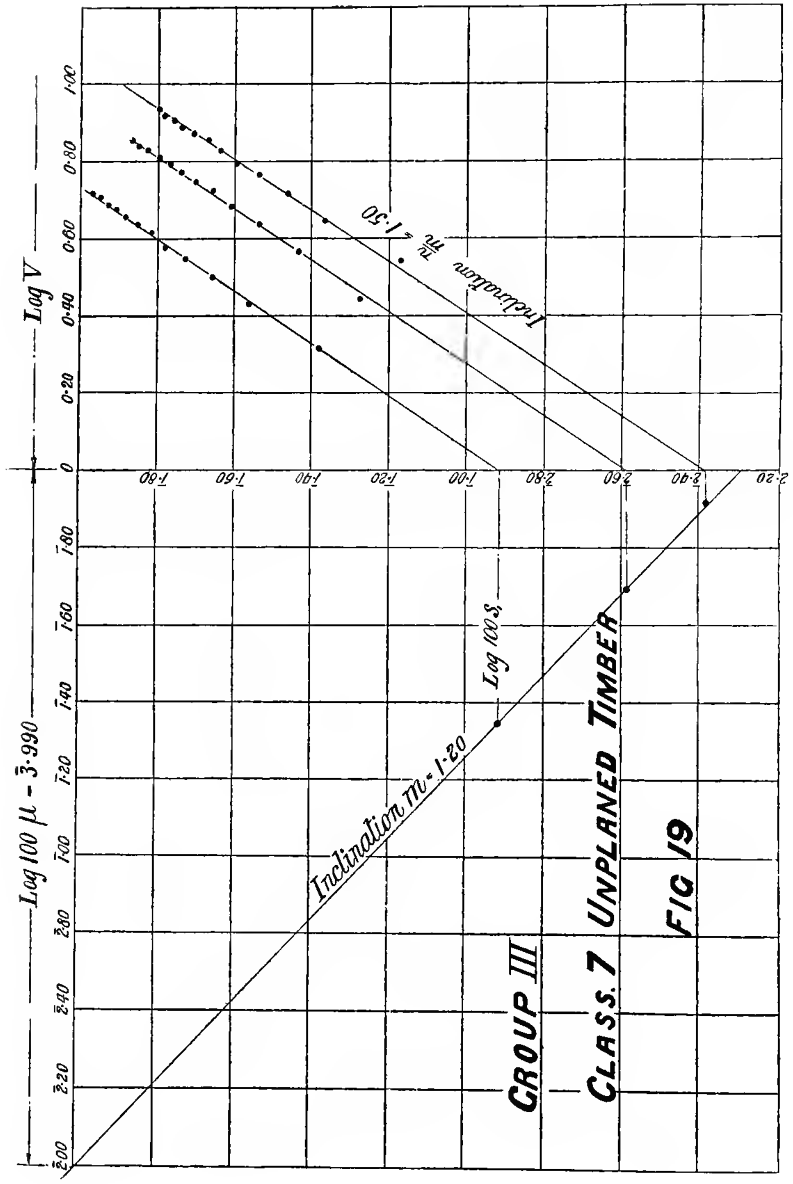
Continuing the three lines until they intersect the axes OX, OZ in the points P_1, Q_1, etc., the lengths Q_1T_1, Q_2T_2, Q_3T_3 are set off upon the logarithmic scale, and represent the negative logs. of s_1, s_2, and s_3, giving three points T_1, T_2, T_3, which again are found to lie in one straight line WY, whose inclination $\frac{OW}{OY}$ fixes the value of m at 1·80, whence it follows, of course, that $n = 120$, while the length OW gives the quantity log. 1000 μ, and reading it off upon the squared paper, we find that log. 1000 $\mu = \bar{2}{\cdot}990$, or log. $\mu = \bar{5}{\cdot}990$, so that $\mu = 0{\cdot}0000978$ nearly.

The constants are determined with a fair degree of accuracy, and each one of them is found to be somewhat greater than the corresponding figure deduced for the smoothest pipes; the constant m shows an increase as well as n, and the ratio $\frac{n}{m}$ is unaltered.[1]

The resistance of this timber surface appears to be very nearly the same as that of the ordinary brickwork (Series 142–153) and of the ashlar masonry example placed in the same group in Table 8.

A distinctly rougher surface and higher resistance are exemplified in the next group, by the three series of experiments 158–164, 165–171, and 172–178, the channel being roughened for this purpose by cross-strips fixed at close intervals of $\frac{3}{8}$ inch (not much wider or deeper than an open mortar joint). When the results of this series are plotted upon the diagram, the points lie

[1] The experiments include six other series of gaugings in the timber channel. Apart from a few aberrations, which may be attributed to the ordinary errors of observation, they serve only to confirm the values above found.


Log V
Log 100 μ - 3·990
Inclination m/n = 1·50
Inclination m = 1·20
Log 100 S
GROUP III
CLASS 7 UNPLANED TIMBER
FIG 19

in three straight lines which are nearly, though not quite, parallel; and they clearly indicate a further increase in all the constants. The value of n is increased to at least 1·90, and m to about 1·33, the ratio ϕ being somewhat less than before.

Closely allied to this example, on the scale of resistance, we have the experimental channel of the next series, in which the internal surface was lined with small pebbles; and a little lower may be placed the sluice-channel of rough hammer-dressed masonry referred to in the two series 201–204 and 205–208. These results admit of the same graphic treatment, and they appear to give slightly higher values for n and m, with a somewhat greater value of μ.

A much higher resistance was obtained when M. Bazin roughened the surface by cross-strips placed at wider intervals of 2 inches; and, plotting the observations of Series 209–215, 216–222, and 223–229, we accordingly find a further increase in all the constants. The value of n is not less than 2·1; while m has increased to about 1·50, and the ratio ϕ, if it has been altered at all, has again been slightly reduced.

With this high grade of roughness may be classed the lining of coarse pebbles ($1\frac{1}{4}$ to 2 inches in diameter) in Bazin's experimental channel, and the three examples of dry-rubble masonry placed in the same group in Table 11.

It is difficult to obtain any consistent results from the scattered examples of still greater roughness; but in many cases they seem to indicate a further increase in all the constants, the value of n being often greater than 2·0, while m increases always with n.

Turning to the opposite end of the scale, the Sudbury conduit of smooth brick appears to range itself pretty nearly along with the smoothest of the lines of jointed pipes. From the data given in Series 64–72, we find again that $\phi = 1{\cdot}50$; and using this value for the individual gaugings of the 2nd and 3rd trials, it appears that n is about 1·75, and m not far from $1\frac{1}{8}$, while log. $\mu = \bar{5}{\cdot}896$.

The same constants apply very nearly to the semicircular channel, lined with a rendering of cement and sand.

When the lining is of pure cement the resistance is sensibly reduced; but using the same values of n and m, it is only necessary to make some reduction in the coefficient μ in order to obtain a close agreement with the observed results. For a semicircular

channel, it would appear that log. $\mu = \bar{5}\cdot830$, but the rectangular form shows a greater resistance, bringing log. μ to $\bar{5}\cdot896$ again.

In all cases where a comparison has been made between the circular and the rectangular form, we find a similar difference.

With the aid of more extended observations of a systematic kind, it will, no doubt, be possible to get a more accurate estimate of the three constants, and a more refined graduation of their differences; but, so far as our information goes, we may venture provisionally to group the materials under five heads according to the values of n and m, which appear to be *nearly* uniform throughout the materials of one group. Using round numbers, or easily workable fractions, the constants would stand as follows :—

TABLE 3.—CONSTANTS FOR THE LOGARITHMIC FORMULA.

Group.	Class.	Form.	Material.	n	m	Log. μ
I.	1	Circular	Smoothest pipes	1·75	1·167	$\bar{5}\cdot878$
	2	,,	Smooth neat cement	,,	,,	$\bar{5}\cdot830$
	2A	Rectangle	,, ,, ...	,,	,,	$\bar{5}\cdot896$
	3	Circular	Rendering cement and sand ...	,,	,,	$\bar{5}\cdot896$
	4	,,	Hard smooth brick	,,	,,	$\bar{5}\cdot896$
	5	Rectangle	Very smooth ashlar	,,	,,	$\bar{5}\cdot956$
II.	6	...	Bare metal pipes, riveted joints	1·77	1·18	$\bar{5}\cdot940$
III.	7	Rectangle	Unplaned timber	1·80	1·20	$\bar{5}\cdot990$
	8	Circular	Rough brickwork	,,	,,	$\bar{5}\cdot975$
	8A	Rectangle	,, ,, ...	,,	,,	$\bar{4}\cdot050$
	9	,,	Ordinary ashlar	,,	,,	$\bar{4}\cdot050$
IV.	10	Rectangle	Timber roughened with strips, $\frac{3}{8}$ inch spaces	1·90	1·33	$\bar{4}\cdot100$
	11	Circular	Lined with fine gravel	,,	,,	$\bar{4}\cdot080$
IVA.	12	Rectangle	Hammer-dressed masonry ...	1·96	1·40	$\bar{4}\cdot182$
	11A	,,	Lined with gravel	,,	,,	$\bar{4}\cdot182$
V.	13	Rectangle	Timber roughened with strips, 2 inch spaces	2·10	1·50	$\bar{4}\cdot300$
	14	,,	Lined with coarse gravel ...	,,	,,	$\bar{4}\cdot270$
	15	,,	Rubble masonry	,,	,,	$\bar{4}\cdot350$

The written values of n and m are those which can actually be determined from experiment by means of the diagram, the most

clearly defined being those given for classes 1, 7, 10, and 13. Between these we have materials which exhibit intermediate degrees of frictional resistance, but the experiments are not complete enough to yield independent values of n and m for every grade of material. They will, however, range themselves quite certainly *in the order* in which they are here placed, and we can either collect them into groups, or interpolate for each grade intermediate values of m and n. Thus it is certain that the kind of work illustrated in Classes 8A and 9 must occupy an intermediate position between 7 and 10. If we use for these materials $n = 1{\cdot}80$, and $m = 1{\cdot}20$, then experiment shows that log. μ will be nearly $\bar{4}{\cdot}050$, and, with that coefficient the formula will agree very well with the facts that are known; although it is quite probable that a more extended range of experiment would have indicated for n some value between 1·80 and 1·90.

It is obvious, however, that in any case the figures can only be approximations, and they will only be applicable within the ordinary limits of engineering practice.

They imply, for example, that the difference (or rather the ratio) between the frictional resistances of a smooth and of a rough material will be greatest in small culverts, and least in very large ones—and this is undoubtedly true; but when the diameter is 40 feet, and the velocity only 1 foot per second, the figures indicate that the difference between any two materials becomes a vanishing quantity, and beyond this limit they cannot be strictly accurate.

Again, they imply that, in the case of a 4-foot culvert, the difference depends upon the velocity, and grows less as the current becomes slower—and experiment shows that this is also true,—but when the velocity is only a few inches per second the figures indicate again that the difference should disappear, and beyond this limit they cannot be applicable. Hence, the table cannot be used for currents that are nearly stagnant, nor for tunnels larger than 40 feet in diameter; and it cannot be used for smooth pipes less than about half-inch in diameter, nor for torrents that exceed some very high velocity.

In any case the calculation must be regarded as purely empirical—derived entirely from experiment, and coinciding fairly well with all the quoted observations, *so far as those observations extend.* When it is applied to cases outside those limits, the formula becomes somewhat precarious, like every other expression that can be devised to represent the known facts.

Art. 29 Results of Formula compared with the Results of Experiment.—To test the formula under all the varying conditions of practice, it has been employed to calculate the velocity V in each one of the examples contained in the following tables. The tables include a large number of the best authenticated gaugings in conduits of various kinds, from glass tubes, less than an inch in diameter, to 4-foot pipes of cast-iron or 9-foot culverts lined with brickwork; while the gradients vary from 1 in 10 up to 1 in 20,000, and the velocities from 2 inches per second up to 20 feet per second.

The divergencies are always most perceptible in the very extreme cases, and often in the lowest of a series of velocities; but, with these exceptions, the general agreement is probably as good as could be obtained by any formula of greater complexity.

In the last column of each table will be found the perpetually changing value of the coefficient $c = \sqrt{K}$ in the old Chezy formula, *i.e.* the variable value which must be adopted in each individual case in order to bring the old formula into agreement with experiment.

Art. 30. Tabulated Gaugings.—The experimental results which have been grouped together in each of the following tables are sufficient in each case to trace the relations between *s*, V, and R, for the given class of material; and they are probably the best that could be chosen for that purpose.

They do not, of course, include an exhaustive statement of all the recorded gaugings; but they have been selected on the principle named in Art. 11 and in Art. 21.

The evidences which they afford are practically confirmed by many other gaugings which are not here quoted; and on the other hand it would be easy to find several recorded observations which appear to contravene these evidences.

In such cases the apparent discordance is probably due to some unspecified difference in the actual character of the material, or to some other abnormal condition in the experiment.

For every gauging the quoted authority is given in the table. The radius R is in English feet, and the velocities are in feet per second.

TABLE 4.—GROUP I. SMOOTH PIPES OF CLASS 1.

$n = 1{\cdot}75$; $m = 1{\cdot}167$; $\mu = 0{\cdot}0000755$; log. $\mu = \bar{5}{\cdot}878$.

No.	Experiment.	s	V calculated.	V observed.	c
1	Glass tube (H. Smith)	0·02501	1·968	1·955	89·5
2	$d = 0{\cdot}0764 = \frac{7}{8}$ inch	0·05077	2·949	2·945	94·6
3	$R = 0{\cdot}0191$	0·07530	3·694	3·685	97·2
4	$l = 63{\cdot}9$ feet	0·10206	4·396	4·383	99·3
5		0·12918	5·030	5·009	100·8
6	Wrought iron pipe,	0·02693	2·243	2·220	91·6
7	glazed lining (H.	0·05219	3·275	3·224	95·5
8	Smith)	0·10338	4·839	4·761	100·2
9	$d = 0{\cdot}0873$	0·13064	5·532	5·443	101·9
	$R = 0{\cdot}02182$				
	$l = 60{\cdot}26$ feet				
10	Wrought iron pipe,	0·00066	0·574	0·577	86·4
11	glazed lining	0·00203	1·090	1·171	99·9
12	(D'Arcy VIII.)	0·00629	2·081	2·182	105·7
13	$d = 0{\cdot}2710 = 3\frac{1}{4}$ inches	0·01220	3·040	3·117	108·4
14	$R = 0{\cdot}0678$	0·02285	4·350	4·442	112·9
15	$l = 365$ feet	0·03107	5·182	5·292	115·3
16		0·04070	6·050	6·148	117·1
17		0·07170	8·361	8·438	121·1
18		0·10654	10·490	10·535	124·0
19		0·13880	12·20	12·034	124·1
	Dantzic water-main.				
20	Cast-iron, lined with	0·000594	1·592	1·577	110·5
21	smooth varnish	0·001376	2·574	2·479	114·1
22	(Lampe)	0·001630	2·836	2·709	114·6
23	$d = 1{\cdot}373 = 16\frac{1}{2}$ ins.	0·001950	3·142	3·090	119·4
	$R = 0{\cdot}3432$				
	$l = 25000$ to 32000 ft.				
	Rosemary pipe, Sudbury.				
24	Cast-iron lined with	0·000318	2·274	2·615	146·7
25	composition (F. P.	0·0007115	3·603	3·738	135·4
26	Stearns)	0·001221	4·906	4·965	142·1
27	$d = 4{\cdot}0$ feet	0·001849	6·218	6·195	144·1
	$R = 1{\cdot}00$				
	$l = 1747$ feet				

NOTE.—Gauging No. 24 is not relied on by Mr. H. Smith, who describes it as a palpable error.

TABLE 5.—GROUP I. CONDUITS WITH SMOOTH LINING OF NEAT CEMENT. CLASSES 2 AND 2A, CIRCULAR AND RECTANGULAR SECTION.

$n = 1{\cdot}75$; $m = 1{\cdot}167$.

No.	Experiment.	R	V calc.	V obs.	c
28	Open channel of semicircular	0·366	3·007	3·02	128·9
29	section (Bazin)	0·503	3·717	3·72	135·6
30	diameter, 4·10 feet	0·605	4·204	4·16	138·0
31	inclination, s = 0·0015	0·682	4·554	4·60	143·7
32		0·750	4·852	4·87	145·1
33	For circular section—	0·809	5·103	5·12	147·1
34	$\mu = 0{\cdot}0000676$;	0·867	5·344	5·29	146·7
35	log. $\mu = \bar{5}{\cdot}830$	0·915	5·539	5·51	148·8
36		0·949	5·676	5·75	152·5
37		0·992	5·846	5·91	153·3
38		1·029	5·991	6·06	154·2
39		1·034	6·010	6·11	155·1
40	Open channel of rectangular	0·168	3·227	3·34	116·5
41	section (Bazin)	0·251	4·217	4·39	125·1
42	width, 5·94 feet	0·322	4·980	5·04	126·9
43	inclination, s = 0·0049	0·375	5·512	5·68	132·4
44		0·430	6·038	6·08	132·4
45	For rectangular section—	0·474	6·443	6·51	135·1
46	$\mu = 0{\cdot}0000787$;	0·518	6·836	6·83	135·5
47	log. $\mu = \bar{5}{\cdot}896$	0·558	7·184	7·12	136·2
48		0·595	7·498	7·41	137·2
49		0·632	7·805	7·63	137·2
50		0·665	8·075	7·86	137·8
51		0·696	8·320	8·07	138·2

TABLE 6.—GROUP 1. CLASSES 3, 4, AND 5. CONDUITS FACED WITH A RENDERING OF CEMENT AND SAND, WITH SMOOTH ASHLAR, OR WITH HARD SMOOTH BRICK.

$n = 1{\cdot}75$; $m = 1{\cdot}167$.

No.	Experiment.	R	V calc.	V obs.	c
52	Open channel of semicircular	0·379	2·822	2·87	120·5
53	section, rendered 2 of	0·529	3·525	3·43	122·0
54	cement to 1 of fine sand	0·635	3·990	3·87	125·3
55	(Bazin)	0·706	4·273	4·30	132·1
56	diameter, 4·10 feet	0·787	4·593	4·51	131·3
57	inclination, $s = 0{\cdot}0015$	0·839	4·794	4·80	135·3
58		0·900	5·023	4·94	134·5
59	For circular section—	0·941	5·175	5·20	138·3
60	$\mu = 0{\cdot}0000787$;	0·983	5·327	5·38	140·1
61	log. $\mu = \bar{5}{\cdot}896$	1·006	5·411	5·48	141·0
62		1·022	5·468	5·55	141·7
63		1·038	5·524	5·66	143·5
64	Sudbury culvert, of hard	1·0779	1·791	1·827	126·7
65	smooth brick, pointed in	1·3723	2·056	2·131	131·6
66	cement (Fteley and	1·3848	2·068	2·139	131·1
67	Stearns)	1·6253	2·301	2·351	133·0
68	Diameter 9·0 feet	1·6282	2·304	2·372	134·0
69	Length 4200 feet	1·8428	2·502	2·564	137·5
70	First trial at uniform in-	2·0485	2·685	2·720	137·2
71	clination $s = 0{\cdot}000192$	2·1916	2·809	2·831	138·9
72		2·3326	2·928	2·926	138·2
	$\mu = 0{\cdot}0000787$;				
	log. $\mu = \bar{5}{\cdot}896$, as above.				

TABLE 6.—GROUP I. CLASSES 3, 4, AND 5—*continued.*

No.	Experiment.	R	s	V calc.	V obs.	c
73	Sudbury culvert, second trial, surface gradient decreased	2·179	0·0000514	1·317	1·432	138·1
74		2·284	0·0000695	1·616	1·716	138·9
75		1·759	0·0001077	1·743	1·820	134·4
76		1·478	0·0001471	1·855	1·912	130·9
77		1·400	0·0001801	2·009	2·071	130·6
78		1·727	0·0001568	2·135	2·198	134·5
79		1·909	0·0001647	2·347	2·406	136·4
80		2·338	0·0001861	2·815	2·909	139·5
81	Third trial, surface gradient increased	1·366	0·0001989	2·091	2·161	130·8
82		1·602	0·0002028	2·350	2·416	133·6
83		1·814	0·0002067	2·583	2·630	135·3
84		2·006	0·0002056	2·753	2·792	137·0
85		2·177	0·0001997	2·860	2·888	138·2
86		2·099	0·0002365	3·074	3·098	137·7
87		2·294	0·0002539	3·397	3·386	138·6
88	Roquefavour aqueduct, smooth brick with floor of cement (D'Arcy). Width 7·4 feet, rectangular, $\mu = 0·0000904$; log. $\mu = \bar{5}·956$	1·504	0·00372	10·98	10·26	137·1
89	Aqueduct de Crau, smooth ashlar (D'Arcy). Width 8·5 feet, retangular, $\mu = 0·0000904$; log. $\mu = \bar{5}·956$	1·774	0·00084	5·24	5·55	125·0

TABLE 7.—GROUP II. CLASS 6. SOCKET PIPES OF BARE CAST IRON OR STONEWARE, AND RIVETED PIPES OF WROUGHT IRON.

$n = 1{\cdot}77$; $m = 1{\cdot}18$; $\mu = 0{\cdot}0000871$; $\log. \mu = \bar{5}{\cdot}940$.

No.	Experiment.	*s*	V calc.	V obs.	*c*
90	Wrought iron pipe	0·00022	0·171	0·205	76·9
91	(D'Arcy)	0·00336	0·793	0·858	82·3
92	$d = 0{\cdot}1296$	0·02389	2·391	2·585	92·9
93	$l = 372$ feet	0·12315	6·014	6·30	99·8
94		0·22408	8·421	8·521	100·0
95	Texas Creek. Riveted pipe of wrought iron (H. Smith) $d = 1{\cdot}416$; $l = 4438$ feet	0·06672	20·97	20·44	131·1
96	New cast-iron pipe	0·00045	1·390	1·380	101·6
97	(D'Arcy)	0·00045	1·390	1·472	108·4
98	$d = 1{\cdot}6404$	0·00060	1·635	1·559	99·4
99	$R = 0{\cdot}4101$	0·00120	2·414	2·602	117·3
100	$l = 365$ feet	0·00125	2·470	2·609	115·2
101		0·00210	3·307	3·416	116·4
102		0·00230	3·476	3·653	119·0
103		0·00260	3·729	3·674	112·5
104		0·00250	3·648	3·700	115·6
105	Stoneware socket pipe (Bidder) $d = 1{\cdot}50$; $R = 0{\cdot}375$	0·00250	3·437	3·581	117·0

TABLE 8.—GROUP III. CLASSES 7, 8, AND 9. OPEN CHANNEL OF UNPLANED TIMBER, RECTANGULAR AQUEDUCTS OF ORDINARY BRICKWORK AND ASHLAR.

$n = 1{\cdot}80$; $m = 1{\cdot}20$.

No.	Experiment.	R	V calc.	V obs.	c
106	Experimental rectangular	0·240	2·161	2·08	93·2
107	channel of unplaned timber	0·363	2·782	2·69	97·8
108	(Bazin)	0·453	3·225	3·16	102·8
109	Width = 6·53 feet	0·528	3·572	3·53	106·5
110	Inclination, $s_1 = 0{\cdot}00208$	0·601	3·805	3·78	106·9
111		0·648	4·094	4·13	112·5
112	For rectangular section	0·704	4·327	4·34	113·5
113	$\mu = 0{\cdot}0000978$;	0·759	4·550	4·51	113·5
114	log. $\mu = \bar{5}{\cdot}990$	0·801	4·716	4·72	115·8
115		0·846	4·891	4·88	116·3
116		0·880	5·021	5·09	119·0
117		0·922	5·180	5·21	118·9
118	Same channel, second trial	0·188	2·89	2·71	89·3
119		0·272	3·70	3·70	101·2
120	Inclination $s_2 = 0{\cdot}0049$	0·342	4·31	4·35	106·2
121		0·402	4·80	4·85	109·4
122		0·453	5·19	5·29	112·2
123		0·504	5·57	5·61	113·0
124		0·547	5·89	5·93	114·5
125		0·587	6·17	6·23	116·1
126		0·628	6·45	6·45	116·4
127		0·662	6·69	6·71	117·8
128		0·698	6·92	6·90	117·9
129		0·727	7·12	7·15	119·8

TABLE 8.—GROUP III. CLASSES 7, 8, AND 9—*continued.*

No.	Experiment.	R	V calc.	V obs.	c
130	Same channel, third trial	0·147	3·27	3·52	101·4
131		0·231	4·42	4·42	101·4
132	Inclination, $s_3 = 0·00824$	0·289	5·14	5·23	107·1
133		0·341	5·74	5·83	109·8
134		0·393	6·30	6·24	109·7
135		0·431	6·70	6·74	113·1
136		0·466	7·06	7·17	115·8
137		0·506	7·46	7·44	115·2
138		0·541	7·80	7·73	115·8
139		0·572	8·09	8·03	116·9
140		0·604	8·39	8·26	117·1
141		0·630	8·63	8·57	119·0
142	Rectangular channel of ordi-	0·192	2·71	2·75	89·7
143	nary brickwork, rather	0·284	3·52	3·66	98·3
144	rough (Bazin)	0·365	4·16	4·18	98·8
145	width = 6·27 feet	0·424	4·60	4·72	103·7
146	inclination, $s = 0·0049$	0·481	5·00	5·10	105·1
147		0·540	5·40	5·33	103·7
148	For rectangular section—	0·582	5·68	5·68	106·3
149	$\mu = 0·0001122$; $\log. \mu = \overline{4}·050$	0·620	5·93	6·01	109·0
150		0·668	6·23	6·15	107·4
151		0·697	6·41	6·47	110·8
152		0·739	6·65	6·60	109·7
153		0·779	6·90	6·72	108·7
154	Chazilly Canal, of ashlar	0·41	5·51	5·73	100·0
155	masonry (D'Arcy)	0·57	7·41	7·52	111·0
156	Inclination, $s = 0·0081$	0·68	8·33	8·19	110·0
157	$\mu = 0·0001122$; $\log. \mu = \overline{4}·050$	0·77	9·05	8·75	111·0

TABLE 9.—GROUP IV. CLASSES 10 AND 11.

$n = 1{\cdot}90$; $m = 1{\cdot}33$.

Experimental Timber Channel with Roughened Linings.

No.	Experiment.	R	V calc.	V obs.	c
158	Timber channel with cross	0·302	1·59	1·65	77·4
159	strips, 2 inches wide, $\frac{3}{8}$ inch	0·442	2·08	2·17	84·5
160	apart (Bazin)	0·634	2·68	2·86	91·0
161		0·775	3·08	3·33	94·0
162	Width, 6·43 feet	0·889	3·39	3·68	97·0
163	Inclination, $s_1 = 0{\cdot}0015$	0·986	3·65	3·98	99·0
164	$\mu = 0{\cdot}000126$; log. $\mu = \bar{4}{\cdot}100$	1·076	3·85	4·19	99·0
165	The same channel, laid at new	0·205	2·50	2·50	71·8
166	inclination, $s_2 = 0{\cdot}0059$	0·302	3·27	3·34	79·0
167		0·442	4·27	4·40	86·2
168		0·552	5·00	5·08	89·0
169		0·643	5·56	5·63	91·4
170		0·716	6·00	6·14	94·5
171		0·790	6·42	6·48	94·8
172	The same channel, laid at the	0·182	2·85	2·85	70·8
173	inclination $s_3 = 0{\cdot}00886$	0·273	3·78	3·75	76·4
174		0·403	4·97	4·92	82·4
175		0·499	5·77	5·77	86·8
176		0·582	6·42	6·38	88·9
177		0·658	7·00	6·86	89·9
178		0·726	7·49	7·26	90·5
179	Semicircular channel, lined	0·454	2·17	2·17	78·0
180	with small pebbles held in	0·546	2·47	2·50	82·0
181	cement (Bazin)	0·619	2·69	2·69	82·0
182		0·681	2·88	2·93	84·0
183	Diameter, 4·0 feet	0·731	3·03	3·05	84·0
184	Inclination, $s = 0{\cdot}0015$	0·784	3·18	3·22	85·0
185		0·826	3·30	3·33	84·0
186	For circular form—	0·900	3·50	3·54	85·0
187	$\mu = 0{\cdot}000120$; log. $\mu = \bar{4}{\cdot}080$	0·968	3·69	3·73	85·0
188		1·012	3·81	3·95	88·0

TABLE 10.—GROUP IVA. CLASSES 11A AND 12.

$n = 1{\cdot}96$; $m = 1{\cdot}40$; log. $\mu = \bar{4}{\cdot}182$.

No.	Experiment.	R	V calc.	V obs.	c
189	Rectangular channel, lined	0·250	2·19	2·16	61·7
190	with pebbles $\frac{3}{8}$ inch to $\frac{7}{8}$	0·357	2·82	2·95	70·5
191	inch diameter (Bazin)	0·450	3·33	3·40	72·5
192		0·520	3·70	3·84	76·1
193	Width, 6·01 feet	0·588	4·03	4·14	77·2
194	inclination, $s = 0{\cdot}0049$	0·644	4·30	4·43	78·8
195		0·700	4·57	4·64	79·3
196	For rectangular form—	0·746	4·78	4·88	80·7
197	$\mu = 0{\cdot}000152$; log. $\mu = \bar{4}{\cdot}182$	0·785	4·95	5·12	82·6
198		0·832	5·16	5·26	82·4
199		0·871	5·34	5·43	83·1
200		0·910	5·51	5·57	83·4
201	Rectangular sluice channel,	0·324	12·31	12·29	67·9
202	865 feet long, of hammer-	0·467	16·00	16·18	74·5
203	dressed masonry (D'Arcy	0·580	18·66	18·68	77·2
204	and Bazin)	0·662	20·52	21·09	81·6
	Inclination of first part				
	$s_1 = 0{\cdot}101$				
205	Second part of the same	0·424	8·94	9·04	72·2
206	channel	0·620	11·72	11·46	75·7
207	Inclination, $s_2 = 0{\cdot}037$	0·745	13·37	13·55	81·6
208	Width throughout = 5·91 feet	0·852	14·72	15·08	84·9

TABLE 11.—GROUP V. CLASSES 13, 14, AND 15.

$n = 2{\cdot}10$; $m = 1{\cdot}50$.

Timber Channel with Roughened Linings, and Channels in Rubble Masonry.

No.	Experiment.	R	V calc.	V obs.	c
209	Rectangular timber channel	0·378	1·29	1·28	53·7
210	with cross-strips 2 inches	0·550	1·70	1·68	58·6
211	apart (Bazin)	0·777	2·18	2·21	64·8
212		0·942	2·51	2·55	67·8
213	Width, 6·43 feet	1·073	2·75	2·81	70·1
214	First inclination, $s_1 = 0{\cdot}0015$	1·197	2·97	2·97	70·0
215		1·299	3·08	3·11	70·5
216	Same channel	0·264	1·94	1·91	48·3
217	Second inclination $s_2 = 0{\cdot}0059$	0·384	2·53	2·56	53·7
218		0·553	3·29	3·37	59·0
219		0·686	3·83	3·88	61·0
220		0·791	4·25	4·31	63·0
221		0·882	4·59	4·65	64·5
222		0·965	4·89	4·91	65·1
223	Same channel	0·232	2·15	2·21	48·7
224	Third inclination, $s_3 = 0{\cdot}00886$	0·350	2·88	2·85	51·2
225		0·509	3·76	3·75	55·8
226	$\mu = 0{\cdot}00020$; log. $\mu = \bar{4}{\cdot}300$	0·628	4·37	4·37	58·6
227	throughout	0·725	4·84	4·85	60·5
228		0·812	5·25	5·22	61·5
229		0·885	5·58	5·57	62·9

TABLE 11.—GROUP V. CLASSES 13, 14, AND 15—*continued.*

$n = 2{\cdot}10$; $m = 1{\cdot}50$.

No.	Experiment.	R	V calc.	V obs.	c
230	Rectangular channel lined with coarse pebbles, $1\frac{1}{4}$ to 2 inches diameter (Bazin)	0·291	1·96	1·79	47·5
231		0·417	2·54	2·43	53·8
232		0·510	2·93	2·90	58·0
233		0·587	3·25	3·27	61·1
234	Width, 6·11 feet	0·656	3·52	3·56	62·8
235	Inclination, $s = 0{\cdot}0049$	0·712	3·73	3·85	65·2
236		0·772	4·04	4·03	65·5
237	$\mu = 0{\cdot}000187$; log. $\mu = \bar{4}{\cdot}270$	0·823	4·13	4·23	66·6
238		0·867	4·29	4·43	68·0
239		0·909	4·44	4·60	69·0
240		0·946	4·57	4·78	70·3
241		0·987	4·70	4·90	70·4
	Dry Rubble Masonry $\mu = 0{\cdot}000224$; log. $\mu = \bar{4}{\cdot}350$				
242	Head race, Kapnikbanya, Hungary (Rittinger)	0·213	1·27	1·37	48·1
243	Inclination, $s = 0{\cdot}0038$ (trapezoidal)	0·344	1·80	1·83	50·6
244	Tail race, Staukau (Rittinger)	0·289	1·30	1·26	46·8
245	Inclination, $s = 0{\cdot}0025$	0·359	1·52	1·49	49·8
246	(semicircular)	0·419	1·69	1·64	50·8
247	Aqueduct at Libeth (Rittinger)	0·213	1·38	1·32	42·8
248	Inclination, $s = 0{\cdot}0045$	0·439	2·32	2·39	53·9
249	(rectangular, nearly)	0·486	2·49	2·43	53·0

CHAPTER V.

CALCULATIONS RELATING TO UNIFORM FLOW.

Art. 31. Nature of the Problems.—In the ordinary course of engineering work, calculations will have to be made upon various data, and the unknown quantity may be either s, h, V or Q, and sometimes d or R. Generally the question will take one of the following shapes:—

(1) With a given section of channel, or given diameter of pipe d, it is required to find the inclination or hydraulic gradient s that will suffice to carry off a certain discharge Q, or will ensure a certain velocity V.

(2) When the gradient s is fixed by the circumstances of the case, it will be necessary to calculate the velocity of current or the discharge Q that will be obtained with a pipe of given diameter or channel of given section.

(3) When the gradient is determined beforehand and the maximum flood discharge Q is known, we may have to reverse the last question, and to calculate the requisite diameter of the culvert or pipe, or to find how high the flood-water would rise in a channel of known section.

We may consider how these questions are to be dealt with by the logarithmic method, or by the more familiar method of the old formula.

Art. 32. To find the Gradient for an Open Channel or the Loss of Head in a given Length of Pipe.—If the required gradient s is to be expressed as a decimal, its logarithm will be given by the formula just as it stands:—

$$\log. s = \log. \mu + n \log. \mathrm{V} - m \log. \mathrm{R}$$

Example 1.—A circular culvert or sewer 8 feet in diameter is to be built of hard smooth brick; and to carry off the flood discharge the velocity must be 2·79 feet per second. At what inclination should the sewer be laid?

Here R will be exactly one-fourth of the diameter, or 2 feet; and for Class 4 in Table 3, we may take $n = 1{\cdot}75$, $m = 1\frac{1}{6}$, and $\log. \mu = \bar{5}{\cdot}896$. The calculation will then stand as follows:—

$$
\begin{array}{llr}
 & & \log. \mu = \bar{5}{\cdot}896000 \\
\log. V = \log. (2{\cdot}79) = 0{\cdot}445604 & & \\
\text{Multiply by } \frac{7}{4} \qquad\qquad 7 & & \\
\qquad 4)\overline{3{\cdot}119228} & & \\
\qquad 0{\cdot}779807 = n \log. V & & \\
\text{Add } n \log. V & & = 0{\cdot}779807 \\
 & & \overline{4{\cdot}675807} \\
\log. R = \log. (2{\cdot}0) = 0{\cdot}301030 & & \\
\text{Multiply by } \frac{7}{6} \qquad\qquad 7 & & \\
\qquad 6)\overline{2{\cdot}107210} & & \\
\qquad 0{\cdot}351202 = m \log. R & & \\
\text{Subtract } m \log. R & & = 0{\cdot}351202 \\
\text{and we have } \log. s & & = \bar{4}{\cdot}324605
\end{array}
$$

Therefore $s = 0{\cdot}0002112$

It is sometimes more convenient to express the gradient as a fraction, and the denominator of that fraction can always be directly found by simply writing the negative value of log. s.

Thus in the example above given we have found that log. $s = \bar{4}{\cdot}324605$, which is equivalent to $- 4{\cdot}000000 + 0{\cdot}324605 = - 3{\cdot}675395$: so that $\log. \frac{1}{s} = 3{\cdot}675395$, which is the common logarithm of 4736. Therefore the gradient is 1 in 4736.

In either case the calculation is quite as simple as the working out of the older and more familiar formula $s = \frac{V^2}{KR}$, which would stand as follows, if we take Beardmore's value of $K = 8874$, while $R = 2{\cdot}0$ and $V = 2{\cdot}79$:—

By a table of squares $V^2 = 2{\cdot}79^2 = 7{\cdot}7841$.

Dividing by $R = 2$, and again by $K = 8874$, we should have—

$$s = \frac{7{\cdot}7841}{2 \times 8874} = 0{\cdot}0004386 = 1 \text{ in } 2280.$$

The two methods give widely different results, the calculated fall being twice as great, according to the old method, as the fall that is actually necessary in such a culvert if we may judge by the observations of Messrs. Fteley and Stearns. In the case of

Experiment No. 84 we find V = 2·792, R = 2·006, and the actual gradient is s = 0·0002056 = 1 in 4863.

Whenever the conditions resemble those that have been supposed in this example, the loss of head will be greatly overestimated by the old method.

Example 2.—A cast-iron water main, 6 inches in diameter, is coated with Angus Smith's composition. When the velocity is 0·8 feet per second, what will be the loss of head due to friction?

Neglecting for the present the effect of bends, we may take the constants given in Table 3 for pipes of Class 1, making $n = 1\frac{3}{4}$, $m = 1\frac{1}{6}$, log. $\mu = \bar{5}\cdot878$; but in this example both V and R are less than unity. The logarithm of the *hydraulic mean gradient* $\frac{h_2}{l_2}$ will be log. s = log. $\mu + n$ log. V $- m$ log. R; and the calculation will be most simply effected by taking out the negative logarithms of V and R as follows:—

$$\text{log. } \mu = \bar{5}\cdot878000$$

log. V = log. (0·8) = $\bar{1}\cdot903090$

whose negative value is $-0\cdot096910$

Multiply by $\frac{7}{4}$ 7

$4)\underline{-0\cdot678370}$

$-0\cdot169592$

Add this negative quantity, or subtract $\underline{0\cdot169592}$

$\bar{5}\cdot708408$

R = 0·125 and log. R = $\bar{1}\cdot096910$

whose negative value is $-0\cdot903090$

Multiply by $\frac{7}{6}$ 7

$6)\underline{-6\cdot321630}$

$-1\cdot053605$

Subtract this negative quantity, or *add* $\underline{1\cdot053605}$

and we have   log. $s = \bar{4}\cdot762013$

or s = 0·000578

The negative value of log. s is $-3\cdot237987$

or, gradient = 1 in 1730.

In this particular case the old formula, with Mr. Beardmore's coefficient would give almost exactly the same result. We should have by this method—

$$s = \frac{V^2}{8874} = \frac{0\cdot64 \times 8}{8874} = 0\cdot000577, \text{ or } 1 \text{ in } 1734.$$

But the coincidence would disappear if the diameter or the velocity were considerably greater; and the discrepancy would be very great in the case of a large pipe worked at a high velocity.

To find the loss of head h_2 in any given length of pipe l_2 we have only to multiply the gradient s by the length l_2 measured in feet: or by the logarithmic method we have—

$$\log. h_2 = \log. s + \log l_2.$$

Thus, in the present example, the loss of head upon a length of 1 mile, or 5280 feet, will be given as follows:—

$$
\begin{array}{lrl}
 & \log. s \text{ as above} = & \bar{4}\cdot762013 \\
\text{Add the log. of } 5280 & \text{or} & 3\cdot722634 \\
\hline
 & & 0\cdot484647 = \log. \text{ of } 3\cdot05.
\end{array}
$$

The loss of head is therefore 3·05 feet per mile.

It will generally be found in practice that a table of common logarithms furnishes the readiest means of making *all* the reductions that are required in these calculations, as, for instance, when the fall has to be computed from given values of the discharge Q, the sectional area a and the wetted perimeter p, in which case $R = \frac{a}{p}$, or $\log. R = \log. a - \log. p$; while $V = \frac{Q}{a}$, or $\log. V = \log. Q - \log. a$.

EXAMPLE 3.—A channel of rectangular section, 2 feet wide and 9 inches deep, is to be executed in hammer-dressed masonry. What fall must it have in a length of 120 feet, in order to carry off 810 cubic feet of water per minute?

The first steps would be to find a, p, R, and V. The given discharge, when expressed in cubic feet per second, is $Q = 810 \div 60 = 13\cdot50$.

The sectional area is $a = 2\cdot0 + 0\cdot75 = 1\cdot50$ square feet. Hence the velocity must be—

$$V = \frac{Q}{a} = \frac{13\cdot50}{1\cdot5} = 9 \text{ feet per second}$$

The wetted perimeter will be equal to the width of floor added to the height of each side wall, or—

$$p = 2\cdot0 + 0\cdot75 + 0\cdot75 = 3\cdot50 \text{ feet}$$

Hence the hydraulic mean depth, or hydraulic radius, will be—

$$R = \frac{a}{p} = \frac{1\cdot50}{3\cdot50} = 0\cdot428 \text{ feet}$$

Then, by the old formula, we should have the required gradient—

$$s = \frac{V^2}{8874R} = \frac{9 \times 9}{8874 \times 0{\cdot}428} = 0{\cdot}02125 = 1 \text{ in } 47$$

and the required fall on the length of 120 feet would be—

$$h_2 = sl_2 = 120 \times 0{\cdot}02125 = 2{\cdot}55 \text{ feet.}$$

To apply the more accurate logarithmic method, we may take from Table 3 the constants for hammer-dressed masonry, Class 12, making—

$$n = 1{\cdot}96,\ m = 1{\cdot}40, \text{ and log. } \mu = \bar{4}{\cdot}182.$$

The calculation would then be carried out as follows :—

		log. μ = $\bar{4}{\cdot}182000$
$Q = 13{\cdot}5$, and log. Q =	$1{\cdot}130334$	
$a = 1{\cdot}5$, and log. a =	$0{\cdot}176091$	
Subtracting, we have log. V =	$0{\cdot}954243$	
Multiply by $1{\cdot}96$	$1{\cdot}96$	
	$1{\cdot}870316 = n$ log. V	
Add n log. V		$1{\cdot}870316$
		$\bar{2}{\cdot}052316$
log. a, as above =	$0{\cdot}176091$	
$p = 3{\cdot}50$, and log. p =	$0{\cdot}544068$	
Subtracting, we have log. R =	$\bar{1}{\cdot}632023$	
whose negative value is	$-0{\cdot}367977$	
Multiply by $1{\cdot}4$	$1{\cdot}4$	
	$-0{\cdot}515168 = m$ log. R	
Subtract this negative quantity, or add		$0{\cdot}515168$
which gives log. s		= $\bar{2}{\cdot}567484$

Whence $s = 0{\cdot}0369$, or 1 in $27{\cdot}07$

$l_2 = 120$, and, adding log. l_2	= $2{\cdot}079181$
we find log. h_2	= $0{\cdot}646665$

Whence the fall is $h_2 = 4{\cdot}43$ feet.

Again, it will be seen that there is a very wide disagreement between the two methods, but, in this case, it is in the opposite direction to the divergence shown in Example 1. The old rule

brings out the fall at not much more than half the value, as estimated by the logarithmic method. The latter estimate is, however, not in excess of the truth, if we may judge by D'Arcy and Bazin's experiments (Nos. 201 to 208), and many others. In the individual gauging of Experiment No. 205, the conditions assumed in our example are nearly reproduced. With a velocity of 9·04 feet per second, and a radius $R = 0·424$, the actual gradient was $s = 0·037$, or 1 in 27 nearly; so that the fall on a length of 120 feet was about 4·44 feet.

If the old method is employed with any other constant value for the coefficient, its results will only approximate to the truth in a few individual cases, while, in others, the calculated loss of head will be either much too great, or just as much too small. For new cast-iron pipes the *average* value of the coefficient is more like $c = 100$, or $K = 10{,}000$, and, for rough and ready calculation, this value is as good as any other; but it will often lead to an error of 20 per cent. in the discharge, or an error of 40 or 50 per cent. in the calculated fall.

Art. 33. To find the Velocity of the Current or the Discharge in Cubic Feet per Second.—Very frequently the gradient is known beforehand, being determined, perhaps, by local conditions; and when the dimensions of the conduit are known, or hypothetically assumed, the problem will be to calculate the velocity of the current, from which it will, of course, be easy to find the discharge in any required units—such as cubic feet per second, or millions of gallons per day of 24 hours.

According to the method which has usually been employed, the mean velocity is calculated by the old formula $V = \sqrt{KRs}$, while the discharge is given by $Q = aV$.

On the other hand, if we use the logarithmic method the formula may be written:—

$$\log. V = \frac{m}{n} . \log. R + \frac{1}{n}(\log. s - \log. \mu)$$

and the computation will be somewhat less tedious than taking out the square root of the product KRs.

Example 1.—A long rectangular trough of sawn timber, 4 feet wide, is laid at the uniform gradient $s = 0·0049$ (or 1 in 204). When the water runs at a uniform depth of 1 foot, what will be the velocity?

It is first necessary to find R; and as the sectional area is

$4 \times 1 = 4$ square feet, while the wetted perimeter is $4 + 1 + 1 = 6$ feet, we have—

$$R = \frac{4 \cdot 0}{6 \cdot 0} = 0 \cdot 667.$$

Inserting this value in the old formula, and using again $K = 8874$, we should find—

$$V = \sqrt{8874 \times \tfrac{2}{3} \times 0 \cdot 0049} = \sqrt{28 \cdot 99} = 5 \cdot 38 \text{ feet per second,}$$

and the discharge would be —

$$Q = 5 \cdot 38 \times 4 \cdot 0 = 21 \cdot 52 \text{ cubic feet per second.}$$

To make the same calculations by the logarithmic formula, we may take, for Class 7 in Table 3, the values $n = 1 \cdot 8$, $m = 1 \cdot 2$, and log. $\mu = \bar{5} \cdot 990$; so that $\frac{m}{n} = \frac{2}{3}$, and $\frac{1}{n} = \frac{5}{9}$.

Then the logarithm of V and of Q will be found as follows:—

$s = 0 \cdot 0049$, and log. s	$= \bar{3} \cdot 690196$	
Subtracting log. μ	$= \bar{5} \cdot 990000$	
we have (log. s − log. μ)	$= 1 \cdot 700196$	
Multiply by $\frac{5}{9}$	5	
	$9)8 \cdot 500980$	
	$0 \cdot 944553$	$0 \cdot 944553$
$R = 0 \cdot 6667$, and log. R	$= \bar{1} \cdot 823909$	
whose negative value is	$-0 \cdot 176091$	
multiply by $\frac{2}{3}$	2	
	$3) -0 \cdot 352182$	
	$-0 \cdot 117394$	
Add this negative quantity, or subtract		$0 \cdot 117394$
which gives log. V		$= 0 \cdot 827159$
So that $V = 6 \cdot 717$ feet per second		
$a = 4 \cdot 00$ square feet, and adding log. a		$= 0 \cdot 602060$
we get log. Q		$= 1 \cdot 429219$

So that $Q = 26 \cdot 87$ cubic feet per second.

This result agrees with Bazin's experiments, the conditions being nearly the same as in his Experiment No. 127, where $s = 0 \cdot 0049$, $R = 0 \cdot 662$, and V was found to be $6 \cdot 71$.

In calculating the discharge of any circular pipe or culvert which is running full, we may remember that its sectional area will be $a = \frac{\pi}{4} . d^2$; and, as the hydraulic radius R is one-fourth of the diameter d, we may write $a = 4\pi R^2 = 12·566R^2$. The logarithm of 12·566 is 1·099209; hence we can express the discharge Q (in cubic feet per second) by—

$$Q = Va = 12·566VR^2$$
$$\text{or, log. } Q = \text{log. } V + 2 \text{ log. } R + 1·099209$$

We may also note that a supply of 1 cubic foot per second is equivalent to 538,440 gallons per day of 24 hours; and the logarithm of this number is 5·731137. The daily supply of a gravitation main will, therefore, be given in gallons by—

$$Q_g = 538440 \ Q = 6766220 \ VR^2$$
$$\text{log } Q_g = \text{log. } V + 2 \text{ log. } R + 1·099209 + 5·731137$$
$$= \text{log. } V + 2 \text{ log. } R + 6·830346$$

In the case of an open semicircular section, running full up to the diametral line, the sectional area will be exactly half as great, or $a = 2\pi R^2 = 6·283 \ R^2$; therefore—

$$Q = 6·283 \ VR^2, \text{ or}$$
$$\text{log. } Q = \text{log. } V + 2 \text{ log. } R + 0·798179$$

and for the daily discharge in gallons—

$$Q_g = 538440 \ Q = 3,383,110 \ VR^2$$
$$\text{log. } Q_g = \text{log. } V + 2 \text{ log. } R + 6·529316$$

Example 2.—An inverted syphon upon a line of gravitation main is laid with 42-inch socket-jointed pipes coated with composition, and the hydraulic mean gradient is 1 in 700. What will be the discharge?

The constants for Class 1 are, as before, $n = 1·75$, $m = 1·167$, and log. $\mu = \bar{5}·878$; so that $\frac{m}{n} = \frac{2}{3}$, and $\frac{1}{n} = \frac{4}{7}$; and the formula will be—

$$\text{log. } V = \tfrac{2}{3} \text{ log. } R + \tfrac{4}{7} (\text{log. } s - \text{log. } \mu)$$

$s = \frac{1}{700}$; and log. s	=	$\bar{3}\cdot154902$	
Subtracting log. μ	=	$\bar{5}\cdot878000$	
we have (log. s − log. μ)	=	$1\cdot276902$	
Multiply by $\frac{4}{7}$		4	
		$7)5\cdot107608$	
		$0\cdot729658$	$0\cdot729658$
R = $\frac{3\cdot5}{4}$ = 0·875; log. R	=	$\bar{1}\cdot942008$	
whose negative value is		$-0\cdot057992$	
Multiply by $\frac{2}{3}$		2	
		$3) -0\cdot115984$	
		$-0\cdot038661$	
Add this negative quantity, or subtract			$0\cdot038661$
and we obtain log. V			$= 0\cdot690997$

So that V = 4·909

The negative value of log. R is	$-0\cdot057992$	
and of 2 log. R	$-0\cdot115984$	
Add this negative quantity, or subtract		$0\cdot115984$
		$0\cdot575013$
then, adding the log. of 12·566, or		$1\cdot099209$
we obtain log. Q		$= 1\cdot674222$

So that Q = 47·23 cubic feet per second.

Or, if we add again,	$5\cdot731137$
we have log. Q_g	$= 7\cdot405359$

So that Q_g = 25,430,000 gallons per day.

It may be well to remember that this estimate would only apply to the smoothest class of pipes, well laid, well jointed, and free from all rust, sediment, or other obstructions.

Art. 34. To find the Diameter of a Circular Pipe or Culvert, when the Discharge Q and the Gradient s are given.—This question is continually presenting itself in the ordinary practice of hydraulic engineering. Its solution by means of the old formula is, however, a somewhat tedious process, while the result is not free from the contingency of considerable error.

The expression that has just been used in Art. 33 for calculating the discharge Q may obviously be written—

$$\log. Q = 1\cdot099209 + \left(2 + \frac{m}{n}\right) \log. R + \frac{1}{n} (\log. s - \log. \mu)$$

whence it follows that R may be determined from the formula—

$$\log. R = \frac{n}{2n+m}\left\{\log. Q - 1{\cdot}099209 + \frac{1}{n}(\log. \mu - \log. s)\right\}$$

and when R has been found, we shall have $d = 4R$.

EXAMPLE 1.—The hydraulic mean gradient upon a certain length of inverted syphon will be 1 in 600. If the syphon consists of a single line of pipe (of Class 1), what must be its internal diameter in order that it may discharge 21 cubic feet per second?

The constants for Class 1 are $n = 1{\cdot}75$, $m = 1{\cdot}167$, and $\log. \mu = \bar{5}{\cdot}878$; so that $\frac{m}{n} = \frac{2}{3}$, and therefore $\frac{n}{2n+m} = \frac{3}{8}$, while $\frac{1}{n} = \frac{4}{7}$.

Hence, for pipes of Class 1, the formula becomes—

$$\log. R = \tfrac{3}{8}\{\log. Q - 1{\cdot}099209 + \tfrac{4}{7}(\log. \mu - \log. s)\}$$

In this example—

$Q = 21{\cdot}0$; and log. Q			$= 1{\cdot}322219$
Subtract the logarithm			$1{\cdot}099209$
			$0{\cdot}223010$
Log. μ, by table	$=$	$\bar{5}{\cdot}878000$	
To subtract log. s we may add log. $\frac{1}{s}$			
$\frac{1}{s} = 600$, and log. $\frac{1}{s}$	$=$	$2{\cdot}778151$	
		$\bar{2}{\cdot}656151$	
whose negative value is		$-1{\cdot}343849$	
Multiply by $\frac{4}{7}$		4	
		$7)-5{\cdot}375396$	
		$-0{\cdot}767914$	
Add this negative quantity, or subtract			$0{\cdot}767914$
			$\bar{1}{\cdot}455096$
The negative value of this is			$-0{\cdot}544904$
Multiply by $\frac{3}{8}$			3
			$8)-1{\cdot}634712$
			$-0{\cdot}204339$
The logarithm of R will therefore be			$\bar{1}{\cdot}795661$

and $R = 0{\cdot}625$

while $d = 4 \times 0{\cdot}625 = 2{\cdot}50$ feet or 30 inches,

which is the required diameter of the pipe.

This would only be sufficient if the pipe were of the smoothest class. For a pipe of Group II. the constants would be higher, and the diameter would come out at about $31\frac{1}{8}$ inches.

If the calculations were made upon the basis of the old formula $V = \sqrt{KRs}$, we should have for the discharge—

$$Q = Va = \frac{\pi}{4}d^2\sqrt{KRs} = \frac{\pi}{4}d^2\sqrt{\frac{K}{4}ds}$$

To obtain the diameter d we should have to square both sides of the equation, or—

$$Q^2 = \frac{\pi^2}{16}d^4\frac{K}{4}ds = \frac{\pi^2}{64} \quad Ksd^5$$

$$\text{whence } d = \sqrt[5]{\frac{64Q^2}{Ks\pi^2}} \qquad (15)$$

Of course the computation could only be made in practice by the aid of a table of logarithms, and the formula would be —

$$\log. d = \tfrac{2}{5} \log. Q - \tfrac{1}{5}(\log. s + \log. K) + 0{\cdot}811880$$

When the circular section is used for all the sewers and pipes of an extensive system of drainage, the diameter will have to be separately determined for each constituent portion of the system. The main lines will receive at successive points the tributary streams from the branches; and the large branches in like manner from the smaller ramifications of the system. The diameter will decrease from the outlet upwards, as in the trunk, the branches and the twigs of a spreading tree, each short length being adapted to its own particular discharge Q and its own gradient s. In such a case, therefore, the calculation may have to be repeated many scores or hundreds of times.

Art. 35. The Calculated Discharge of Circular Pipes.—To lighten the labour of repeated computation the hydraulic engineer will often find it convenient to refer to some tables of calculated discharges, and such tables are especially useful when the conditions of the problem are such that both the gradient and the diameter are left to arbitrary adjustment within certain limits. A provisional determination of one or the other can then be made by mere inspection of the table.

For such purposes the following table have been worked out by the method already described in Art. 33. Table 12 gives the discharge in cubic feet per minute for small pipes varying from

1 inch up to 32 inches in diameter, and with hydraulic gradients varying from 1 in 800 to 1 in 5. The calculations are continued in Table 12A for pipes of larger size up to 6 feet in diameter, with gradients varying from 1 in 2000 to 1 in 10, and the discharge is here given in cubic feet per second.

The figures in each case relate only to new and clean pipes of Class 1 with the smoothest internal surface. For such cases the figures are certainly not beyond the estimate which may be confidently based upon the results of experiment; but they will not be realized if the flow is obstructed by sharp bends, or by imperfect jointing, nor if the surface is roughened by the deposit of any sediment or incrustation; and the discharge will be greatly reduced if the metal is pitted by rust, or if the deposit amounts to anything more than a film.

The figures in this table must, therefore, be taken to represent a maximum and not an average. For surfaces of bare metal, cast-iron, lead, or steel-plate, without any glazed coating, the discharge is generally not greater than would be given by the formula for Group II. When the gradient is 1 in 1000 this amounts to 91 per cent. of the figures given in the table, and when the gradient is 1 in 100 it amounts to 89 per cent. This lower estimate for Group II. would again be applicable only when the pipes are quite new and clean, and would be too high for some riveted pipes in which the flow is impeded by lap-joints or internal butts.

For practical purposes, therefore, it will be safe to take the figures of the table as giving the *greatest* discharge that will have to be dealt with at the outlet of any given pipe; but if the table is used to determine the diameter and fall of the pipe for a given discharge, it will be necessary to take a lower estimate. For a newly-laid water-main of either class it would probably be safe to estimate the discharge at nine-tenths of the quantity given by the formula. But no rule can be given for old pipes, in which it is well known that the discharge will often fall to one-half of the original flow.

TABLE 12.—PIPES OF CLASS I.

GRADIENT.	DIAMETER.						
	1 in.	$1\frac{1}{2}$ in.	2 in.	$2\frac{1}{2}$ in.	3 in.	$3\frac{1}{2}$ in.	4 in.
1 in 5	2·19	6·60	14·22	25·78	41·92	63·23	90·27
„ 6	2·02	5·94	12·81	23·23	37·77	56·97	81·34
„ 7	1·85	5·45	11·73	21·27	34·59	52·17	74·48
„ 8	1·71	5·05	10·87	19·71	32·04	48·34	69·01
„ 9	1·60	4·72	10·16	18·42	29·96	45·19	64·52
„ 10	1·51	4·44	9·57	17·35	28·21	42·55	60·75
„ 15	1·20	3·52	7·59	13·76	22·37	33·75	48·19
„ 20	1·01	2·99	6·44	11·67	18·98	28·63	40·88
„ 25	0·89	2·63	5·67	10·28	16·71	25·21	35·99
„ 30	0·80	2·37	5·11	9·26	15·06	22·71	32·43
„ 35	0·74	2·17	4·68	8·48	13·79	20·80	29·69
„ 40	0·68	2·01	4·33	7·86	12·77	19·27	27·51
„ 45	0·64	1·88	4·05	7·34	11·94	18·02	25·72
„ 50	0·60	1·77	3·81	6·92	11·25	16·96	24·22
„ 60	0·54	1·60	3·44	6·23	10·13	15·30	21·82
„ 70	0·50	1·46	3·15	5·71	9·28	14·00	19·98
„ 80	0·46	1·35	2·92	5·29	8·60	12·97	18·51
„ 90	0·43	1·24	2·73	4·94	8·04	12·12	17·31
„ 100	0·40	1·19	2·57	4·65	7·57	11·41	16·30
„ 110	0·38	1·13	2·43	4·41	7·17	10·81	15·43
„ 120	0·36	1·07	2·31	4·19	6·82	10·29	14·68
„ 130	0·35	1·03	2·21	4·00	6·51	9·83	14·03
„ 140	0·33	0·98	2·12	3·84	6·24	9·42	13·45
„ 150	0·32	0·94	2·04	3·69	6·00	9·05	12·93
„ 160	0·31	0·91	1·96	3·56	5·78	8·73	12·46
„ 170	0·30	0·88	1·90	3·44	5·59	8·43	12·03
„ 180	0·29	0·85	1·84	3·33	5·41	8·16	11·65
„ 190	0·28	0·83	1·78	3·22	5·24	7·91	11·30
„ 200	0·27	0·80	1·73	3·13	5·09	7·68	10·97
„ 250	0·24	0·71	1·52	2·76	4·48	6·76	9·65
„ 300	0·22	0·64	1·37	2·48	4·04	6·09	8·70
„ 350	0·20	0·58	1·26	2·27	3·70	5·58	7·97
„ 400	0·18	0·54	1·16	2·11	3·43	5·17	7·38

DISCHARGE IN CUBIC FEET PER MINUTE.

Diameter. 5 in.	6 in.	7 in.	8 in.	9 in.	10 in.	11 in.	Gradient.
163·7	266·2	401·5	573·3	784·7	1039·0	1340·0	1 in 5
147·5	239·9	361·8	516·6	707·1	936·5	1208·0	„ 6
135·0	219·6	331·2	473·0	647·5	857·5	1106·0	„ 7
125·1	203·5	306·9	438·3	600·0	794·5	1024·0	„ 8
117·0	190·2	287·0	409·8	560·8	742·8	957·8	„ 9
110·1	179·1	270·2	385·8	528·1	699·4	901·8	„ 10
87·3	142·1	214·3	306·0	418·8	554·8	715·3	„ 15
74·1	120·5	181·8	259·6	355·4	470·7	606·8	„ 20
65·2	106·1	160·0	228·6	312·8	414·3	534·2	„ 25
58·8	95·6	144·2	206·0	281·9	373·3	481·4	„ 30
53·8	87·5	132·0	188·6	258·1	341·9	440·8	„ 35
49·9	81·1	122·4	174·7	239·1	316·7	408·4	„ 40
46·6	75·8	114·4	163·3	223·6	296·1	381·8	„ 45
43·9	71·4	107·7	153·8	210·5	278·9	359·5	„ 50
39·6	64·3	97·0	138·6	189·7	251·2	324·0	„ 60
36·2	58·9	88·9	126·9	173·7	230·0	296·6	„ 70
33·5	54·6	82·3	117·6	160·9	213·1	274·8	„ 80
31·4	51·0	77·0	110·0	150·5	199·3	256·9	„ 90
29·5	48·1	72·5	103·5	141·7	187·6	241·9	„ 100
28·0	45·5	68·6	98·0	134·2	177·7	229·1	„ 110
26·6	43·3	65·3	93·3	127·7	169·1	218·0	„ 120
25·4	41·4	62·4	89·1	122·0	161·5	208·2	„ 130
24·4	39·6	59·8	85·4	116·9	154·8	199·6	„ 140
23·4	38·1	57·5	82·1	112·4	148·8	191·9	„ 150
22·6	36·7	55·4	79·1	108·3	143·4	185·0	„ 160
21·8	35·5	53·5	76·4	104·6	138·5	178·7	„ 170
21·1	34·3	51·8	74·0	101·3	134·1	172·9	„ 180
20·5	33·3	50·2	71·7	98·2	130·0	167·6	„ 190
19·9	32·3	48·8	69·6	95·3	126·2	162·8	„ 200
17·5	28·5	42·9	61·3	83·9	111·1	143·3	„ 250
15·8	25·6	38·7	55·3	75·6	100·1	129·1	„ 300
14·4	23·5	35·4	50·6	69·2	91·7	118·2	„ 350
13·4	21·8	32·8	46·9	64·2	85·0	109·5	„ 400

TABLE 12.—PIPES OF CLASS I.

GRADIENT.	DIAMETER.						
	12 in.	13 in.	14 in.	15 in.	16 in.	17 in.	18 in.
1 in 10	1137·0	1408·0	1715·0	2062·0	2450·0	2879·0	3353·0
„ 15	902·0	1117·0	1360·0	1635·0	1943·0	2284·0	2660·0
„ 20	765·4	947·4	1154·0	1388·0	1648·0	1938·0	2256·0
„ 25	673·7	834·0	1016·0	1222·0	1451·0	1706·0	1986·0
„ 30	607·1	751·5	915·4	1101·0	1307·0	1537·0	1790·0
„ 35	555·9	688·1	838·3	1008·0	1197·0	1407·0	1639·0
„ 40	515·0	637·6	776·7	933·8	1109·0	1304·0	1519·0
„ 45	481·5	596·1	726·1	873·0	1037·0	1219·0	1420·0
„ 50	453·5	561·3	683·8	822·1	976·5	1148·0	1337·0
„ 60	408·5	505·7	616·0	740·7	879·7	1034·0	1204·0
„ 70	374·1	463·1	564·1	678·2	805·6	947·0	1103·0
„ 80	346·6	429·0	522·6	628·4	746·4	877·4	1022·0
„ 90	324·0	401·1	488·6	587·5	697·8	820·3	955·3
„ 100	305·1	377·7	460·1	553·2	657·0	772·3	899·5
„ 110	289·0	357·6	435·7	523·9	622·2	731·4	852·0
„ 120	274·9	340·3	414·5	498·4	592·0	696·0	810·5
„ 130	262·6	325·1	396·0	476·2	565·5	664·8	774·3
„ 140	251·7	304·5	379·6	456·4	542·1	637·3	742·2
„ 150	242·0	299·6	365·0	438·8	521·1	612·6	713·5
„ 160	233·2	288·7	351·7	422·9	502·3	590·4	687·7
„ 170	225·3	278·9	339·7	408·5	496·5	570·4	664·3
„ 180	218·0	270·0	328·8	395·4	469·6	552·0	642·9
„ 190	211·4	261·7	318·8	383·3	455·3	535·2	623·4
„ 200	205·3	254·2	309·6	372·3	442·1	519·8	605·4
„ 250	180·7	223·7	272·5	327·7	389·2	457·6	532·9
„ 300	162·9	201·6	245·6	295·3	350·7	412·3	480·1
„ 350	149·1	184·6	224·9	270·4	321·1	377·5	439·7
„ 400	138·2	171·0	208·4	250·5	297·5	349·8	407·3
„ 450	129·2	159·9	194·8	234·2	278·2	327·0	380·8
„ 500	121·6	150·6	183·4	220·5	261·9	307·9	358·6
„ 600	109·6	135·7	165·3	198·7	236·0	277·4	323·1
„ 700	100·4	124·2	151·3	182·0	216·1	254·0	300·0
„ 800	93·0	115·1	140·2	168·6	200·2	235·4	274·1

DISCHARGE IN CUBIC FEET PER MINUTE—*continued.*

Diameter.							Gradient.
20 in.	22 in.	24 in.	26 in.	28 in.	30 in.	32 in.	
4441·0	5726·0	7221·0	8940·0	10893·0	13093	15552	1 in 10
3522·0	4542·0	5728·0	7091·0	8640·0	10385	12334	„ 15
2988·0	3853·0	4860·0	6016·0	7330·0	8811	10465	„ 20
2630·0	3392·0	4278·0	5295·0	6453·0	7756	9213	„ 25
2370·0	3056·0	3854·0	4771·0	5814·0	6984	8301	„ 30
2170·0	2799·0	3530·0	4370·0	5324·0	6400	7601	„ 35
2011·0	2593·0	3270·0	4048·0	4933·0	5930	7043	„ 40
1880·0	2424·0	3057·0	3785·0	4612·0	5543	6584	„ 45
1770·0	2283·0	2879·0	3564·0	4343·0	5220	6201	„ 50
1595·0	2010·0	2594·0	3211·0	3913·0	4703	5586	„ 60
1461·0	1883·0	2375·0	2940·0	3583·0	4307	5115	„ 70
1353·0	1745·0	2201·0	2724·0	3319·0	3990	4740	„ 80
1265·0	1631·0	2057·0	2547·0	3103·0	3730	4431	„ 90
1191·0	1536·0	1937·0	2398·0	2922·0	3512	4172	„ 100
1128·0	1455·0	1835·0	2271·0	2767·0	3326	3951	„ 110
1073·0	1384·0	1745·0	2161·0	2633·0	3165	3760	„ 120
1025·0	1322·0	1668·0	2064·0	2515·0	3024	3591	„ 130
983·0	1267·0	1598·0	1979·0	2412·0	2898	3442	„ 140
945·0	1218·0	1537·0	1902·0	2318·0	2786	3309	„ 150
910·7	1174·0	1481·0	1833·0	2234·0	2685	3190	„ 160
879·7	1134·0	1431·0	1771·0	2158·0	2594	3081	„ 170
851·5	1098·0	1385·0	1714·0	2088·0	2510	2982	„ 180
825·6	1064·0	1343·0	1662·0	2025·0	2434	2891	„ 190
801·7	1034·0	1304·0	1614·0	1966·0	2364	2808	„ 200
705·7	910·0	1147·0	1421·0	1731·0	2034	2472	„ 250
636·0	820·0	1034·0	1280·0	1560·0	1875	2227	„ 300
582·3	750·8	947·0	1172·0	1428·0	1717	2039	„ 350
539·5	695·6	877·3	1086·0	1323·0	1590	1890	„ 400
504·4	650·4	820·2	1015·0	1237·0	1487	1766	„ 450
475·0	612·3	772·3	956·0	1165·0	1400	1663	„ 500
427·9	551·8	695·9	861·5	1050·0	1262	1498	„ 600
391·9	505·2	637·2	788·8	961·2	1155	1372	„ 700
363·0	468·1	590·4	730·9	890·4	1070	1272	„ 800

TABLE 12A.—PIPES OF CLASS 1.

GRADIENT.	DIAMETER.						
	34 in.	36 in.	39 in.	42 in.	45 in.	48 in.	51 in.
1 in 10	304·7	354·9	439·3	535·2	643·4	764·2	898·3
„ 20	205·0	238·8	295·6	360·2	433·0	514·3	604·5
„ 30	162·6	189·4	234·5	285·7	343·4	407·9	479·5
„ 40	138·0	160·7	199·0	242·4	291·4	346·0	406·8
„ 50	121·5	141·5	175·1	213·4	256·5	304·7	358·2
„ 60	109·4	127·4	157·8	192·3	231·1	274·5	322·7
„ 70	102·1	119·0	147·3	179·4	215·7	256·2	301·2
„ 80	92·8	108·1	133·9	163·1	196·1	232·9	273·8
„ 90	86·8	101·3	125·2	152·5	183·3	217·7	256·0
„ 100	81·7	95·2	117·8	143·6	172·6	205·0	241·0
„ 110	77·4	90·1	111·6	136·0	163·4	194·1	228·2
„ 120	73·6	85·8	106·2	129·4	155·5	184·7	217·1
„ 130	70·4	81·9	101·4	123·6	148·6	176·5	207·4
„ 140	67·4	78·5	97·2	118·5	142·4	169·1	198·8
„ 150	64·8	75·5	93·5	113·9	136·9	162·6	191·1
„ 160	62·5	72·8	90·1	109·8	132·0	156·7	184·2
„ 170	60·4	70·3	87·0	106·0	127·4	151·4	178·0
„ 180	58·4	68·0	84·2	102·6	123·3	146·5	172·2
„ 190	56·6	66·0	81·7	99·5	119·6	142·1	167·0
„ 200	55·0	64·1	79·3	96·6	116·1	138·0	162·2
„ 250	48·4	56·4	69·8	85·1	102·2	121·4	142·7
„ 300	43·6	50·8	62·9	76·6	92·1	109·4	128·6
„ 350	40·0	46·5	57·6	70·2	84·4	100·2	117·8
„ 400	37·0	43·1	53·4	65·0	78·2	92·8	109·1
„ 450	34·6	40·3	49·9	60·8	73·1	86·8	102·0
„ 500	32·6	38·0	47·0	57·2	68·8	81·7	96·0
„ 600	29·4	34·2	42·3	51·6	62·0	73·6	86·5
„ 700	26·9	31·3	38·8	47·2	56·8	67·4	79·2
„ 800	24·9	29·0	35·9	43·7	52·6	62·5	73·4
„ 900	23·3	27·1	33·6	40·9	49·2	58·4	68·7
„ 1000	21·9	25·5	31·6	38·5	46·3	55·0	64·6
„ 1500	17·4	20·2	25·1	30·5	36·7	43·6	51·2
„ 2000	14·7	17·2	21·3	25·9	31·1	37·0	43·5

DISCHARGE IN CUBIC FEET PER SECOND.

Diameter.							Gradient.
4 ft. 6 in.	4 ft. 9 in.	5 ft.	5 ft. 3 in.	5 ft. 6 in.	5 ft. 9 in.	6 ft.	
1046·0	1208·0	1385·0	1581·0	1786·0	2011·0	2253·0	1 in 10
704·0	813·2	932·4	1064·0	1202·0	1353·0	1516·0	„ 20
558·4	645·0	739·6	844·1	953·6	1073·0	1203·0	„ 30
473·8	547·3	627·5	716·1	809·0	910·9	1020·0	„ 40
417·1	481·8	552·4	630·4	712·3	802·0	898·3	„ 50
375·8	434·1	497·7	568·0	641·7	722·5	809·3	„ 60
350·8	405·1	464·6	530·2	599·0	674·3	755·4	„ 70
318·8	368·3	422·2	481·9	544·4	613·0	686·6	„ 80
298·0	344·3	394·8	450·5	509·0	573·1	642·0	„ 90
280·6	324·2	371·7	424·2	490·4	539·6	604·4	„ 100
265·8	307·0	352·0	401·7	453·9	511·0	572·4	„ 110
252·9	292·1	334·9	382·2	431·8	486·2	544·6	„ 120
241·6	279·0	320·0	365·1	412·5	464·5	520·3	„ 130
231·5	267·5	306·7	350·0	395·4	445·2	498·7	„ 140
222·6	257·1	294·8	336·5	380·1	428·0	479·5	„ 150
214·5	247·8	284·1	324·3	366·4	412·5	462·1	„ 160
207·2	239·4	274·5	313·2	353·9	398·4	446·3	„ 170
200·6	231·7	265·7	303·2	342·5	385·6	432·0	„ 180
194·5	224·6	257·6	294·0	332·1	373·9	418·8	„ 190
188·9	218·1	250·1	285·4	322·5	363·1	406·7	„ 200
166·2	192·0	220·2	251·2	283·9	319·6	358·0	„ 250
149·8	173·0	198·4	226·4	255·8	288·0	322·7	„ 300
137·1	158·5	181·7	207·3	234·2	263·7	295·4	„ 350
127·1	146·8	168·3	192·1	217·0	244·4	273·7	„ 400
118·8	137·2	157·3	179·6	202·9	228·4	255·9	„ 450
111·8	129·2	148·2	169·1	191·0	215·1	241·0	„ 500
100·8	116·4	133·5	152·4	172·1	193·8	217·1	„ 600
92·3	106·6	122·2	139·5	157·6	177·4	198·8	„ 700
85·5	98·8	113·2	129·3	146·0	164·4	184·2	„ 800
80·0	92·3	105·9	120·8	136·5	153·7	172·2	„ 900
75·3	86·9	99·7	113·8	128·5	144·7	162·1	„ 1000
59·7	69·0	79·1	90·2	102·0	114·8	128·6	„ 1500
50·7	58·5	67·1	76·6	86·5	97·4	109·1	„ 2000

Art. 36. Circular Culverts of Brickwork or Ashlar Masonry.—It is certain that the loss of head in a culvert of masonry or brick will depend upon the character of the facework. For the very smooth ashlar of the Aqueduct de Crau, the cement rendering of Bazin's 6-feet semicircle, or the smooth pressed brick of the Sudbury Culvert or the Roquefavour Aqueduct, the resistance is only a little greater than in pipes of Class 1. But the constants come out at considerably higher figures in Group III., where the materials are of a more ordinary character, as in the ashlar-work of the Chazilly Canal and many other examples, in the brickwork of Bazin's rectangular channel and in the circular brick barrel of the Dorchester Bay Tunnel.

For the circular form of the last-named culvert, whose diameter is 7 feet 6 inches, we should naturally expect to find the constant μ somewhat smaller than the value 0·0001122 given for the rectangular section. The gaugings that were taken in this tunnel when it was running full of sea-water give values of log. μ varying from $\overline{5}\cdot952$ to $\overline{4}\cdot001$.[1] Taking the mean value or log. $\mu = \overline{5}\cdot975$ we may probably accept it as representing approximately a lower limit for circular culverts of brick or ashlar masonry; and the actual discharge of any proposed culvert when executed in this class of work, may be expected to range between these limits—its exact value in any individual example being determined by the hardness and smoothness of the material and the character of the workmanship.

The maximum and minimum discharges given in the following table have accordingly been calculated by means of the constants in Table 3 for Class 4 and Class 8 respectively, taking, however, for Class 8, the value log. $\mu = \overline{5}\cdot975$, as found approximately in the Dorchester Bay Tunnel.

There can be little doubt, however, that the discharge will be smaller than the minimum value given in this table, if the ashlar-work is of a rough character, or if the walls are covered with any weedy growth or deposit; and for anything like hammer-dressed or rock-faced masonry the constants must be taken very much higher, as shown in Groups IV. and V. of Table 3.

[1] For the details of these experiments see H. Smith's "Hydraulics," p. 242.

TABLE 13.—CIRCULAR BRICK CULVERTS. DISCHARGE IN CUBIC FEET PER SECOND.

GRADIENT.	DIAMETER.					
	1 foot 6 inches		2 feet		2 feet 6 inches	
	from	to	from	to	from	to
1 in 20	30·00	36·70	64·50	79·0	117·0	143·4
„ 30	23·90	29·10	51·50	62·6	93·4	113·5
„ 40	20·40	24·70	43·90	53·1	79·5	96·4
„ 50	18·00	21·70	38·80	46·8	70·3	84·8
„ 60	16·30	19·60	35·00	42·1	63·5	76·4
„ 70	14·90	17·90	32·90	38·6	58·3	70·0
„ 80	13·50	16·60	29·90	35·8	54·1	64·8
„ 90	13·00	15·50	28·00	33·4	50·7	60·6
„ 100	12·20	14·60	26·40	31·5	47·8	57·1
„ 110	11·60	13·80	25·00	29·8	45·4	54·0
„ 120	11·10	13·20	23·80	28·3	43·2	51·4
„ 130	10·58	12·60	22·80	27·1	41·3	49·1
„ 140	10·16	12·10	21·90	26·0	39·7	47·1
„ 150	9·78	11·60	21·10	25·0	38·2	45·3
„ 160	9·43	11·20	20·30	24·1	36·8	43·6
„ 170	9·12	10·80	19·60	23·2	35·6	42·1
„ 180	8·83	10·40	19·00	22·4	34·5	40·8
„ 190	8·57	10·10	18·50	21·8	33·5	39·5
„ 200	8·33	9·50	18·00	21·2	32·5	38·4
„ 250	7·36	8·66	15·90	18·6	28·8	33·0
„ 300	6·65	7·80	14·40	16·8	26·0	30·5
„ 350	6·10	7·14	13·10	15·4	23·8	27·9
„ 400	5·67	6·62	12·20	14·3	22·2	25·8
„ 450	5·19	6·19	11·40	13·3	20·7	24·2
„ 500	5·01	5·83	10·80	12·5	19·6	22·7
„ 600	4·52	5·25	9·75	11·3	17·7	20·5
„ 700	4·15	4·87	8·95	10·4	16·2	18·8
„ 800	3·86	4·45	8·31	9·6	15·1	17·4
„ 900	3·61		7·78		14·1	
„ 1000	3·41		7·34		13·3	
„ 1500	2·72		5·86		10·6	
„ 2000	2·32		5·00		9·06	

TABLE 13.—CIRCULAR BRICK CULVERTS. DISCHARGE

GRADIENT.	DIAMETER.					
	3 feet		3 feet 6 inches		4 feet	
	from	to	from	to	from	to
1 in 20	190·0	232·9	286·8	351·2	409·5	501·4
„ 30	152·0	184·7	229·0	278·5	327·0	397·7
„ 40	129·4	156·7	195·2	236·3	278·7	337·3
„ 50	114·3	138·0	172·4	208·1	246·1	297·0
„ 60	103·3	124·2	156·0	187·5	222·4	267·6
„ 70	94·8	116·0	143·0	174·9	204·2	249·8
„ 80	88·0	105·4	132·8	159·0	189·5	227·0
„ 90	82·4	98·7	124·4	148·7	177·6	212·2
„ 100	77·8	92·8	117·3	140·0	167·5	200·0
„ 110	73·8	87·9	111·3	132·6	158·8	189·2
„ 120	70·3	83·6	106·0	126·2	151·4	180·1
„ 130	67·2	79·8	101·4	120·5	144·7	172·0
„ 140	64·5	76·5	97·3	115·5	139·0	164·9
„ 150	62·1	73·6	93·6	111·0	133·7	158·5
„ 160	59·9	71·0	90·4	107·0	129·0	152·8
„ 170	57·9	68·6	87·4	103·3	124·7	147·6
„ 180	56·1	66·3	84·6	100·0	120·8	142·8
„ 190	54·5	64·5	82·1	97·0	117·2	138·5
„ 200	52·9	62·5	79·8	94·2	114·0	134·5
„ 250	46·7	55·0	70·5	83·0	100·7	118·3
„ 300	41·9	48·5	63·7	74·7	91·0	106.7
„ 350	38·8	44·9	58·5	68·5	83·5	97·7
„ 400	36·0	42·0	54·3	63·4	77·5	90·5
„ 450	33·7	39·3	50·9	59·1	72·6	84·6
„ 500	31·8	37·0	47·9	55·8	68·5	79·6
„ 600	28·7	33·3	43·4	50·3	61·9	71·7
„ 700	26·4	30·5	39·8	46·0	56·8	65·7
„ 800	24·5	28·3	36·9	42·6	52·7	61·7
„ 900	23·0	26·7	34·6	39·9	49·4	57·0
„ 1000	21·7	24·9	32·7	37·5	46·6	53·6
„ 1500	17·3	19·7	26·1	29·7	37·2	42·5
„ 2000	14·7	16·8	22·2	25·2	31·7	36·1

IN CUBIC FEET PER SECOND—*continued.*

Diameter.						Gradient.
4 feet 6 inches		5 feet		6 feet		
from	to	from	to	from	to	
560·0	686·0	743·0	909·0	1208·0	1478	1 in 20
448·0	544·0	593·0	721·0	964·0	1173	„ 30
381·0	462·0	505·0	612·0	822·0	994	„ 40
337·0	406·0	446·0	538·0	726·0	887	„ 50
305·0	366·0	403·0	485·0	656·0	789	„ 60
279·0	342·0	370·0	453·0	602·0	735	„ 70
260·0	311·0	344·0	411·0	559·0	678	„ 80
243·0	290·0	322·0	385·0	524·0	634	„ 90
229·0	274·0	304·0	362·0	494·0	589	„ 100
217·0	259·0	288·0	343·0	468·0	558	„ 110
207·0	247·0	275·0	326·0	446·0	531	„ 120
198·0	236·0	262·0	312·0	427·0	507	„ 130
190·0	226·0	252·0	299·0	410·0	486	„ 140
183·0	217·0	243·0	287·0	394·0	467	„ 150
177·0	209·0	234·0	277·0	380·0	450	„ 160
171·0	202·0	226·0	267·0	368·0	435	„ 170
165·0	196·0	219·0	259·0	356·0	421	„ 180
160·5	189·5	213·0	251·0	346·0	408	„ 190
156·0	184·0	207·0	244·0	336·0	396	„ 200
138·0	162·0	182·5	215·0	297·0	349	„ 250
124·5	146·0	165·0	193·0	268·0	314	„ 300
114·3	133·7	151·5	177·0	246·0	288	„ 350
106·0	124·0	140·5	164·0	229·0	267	„ 400
99·3	115·8	131·5	153·0	214·0	249	„ 450
93·8	109·0	124·0	144·5	202·0	235	„ 500
84·8	98·3	112·3	130·2	182·0	211	„ 600
77·8	90·0	103·0	119·0	167·5	194	„ 700
72·2	83·4	95·6	110·4	155·5	179	„ 800
67·7	78·0	89·6	103·0	145·7	168	„ 900
63·8	73·4	84·5	97·0	137·5	158	„ 1000
50·9	58·2	67·4	77·0	109·7	125	„ 1500
43·4	49·4	57·5	65·4	93·4	106	„ 2000

Art. 37. Open Channels of Regular Polygonal Form.—In designing the cross-section of an open channel or aqueduct it will always be desirable to adopt such proportions as will ensure the highest velocity of current, and in general this is to be attained by making the ratio $\frac{p}{a}$ as small as possible. In this respect the semicircular form will obviously be the most efficient of all, and among rectilinear forms there are some which are more efficient than others.

Thus the depth of a rectangular channel may bear any proportion to its width, but there is one proportion which is better than any other. The smallest possible ratio of p to a is obtained in this case when the depth is exactly one-half of the width, so that the cross-section of the stream is one-half of a square. Theoretically, therefore, these proportions give the greatest possible discharge for a rectangular channel of any given sectional area; and this is so easily verified that it is unnecessary to give a mathematical demonstration of its truth.

Again, if the aqueduct is to have a flat floor and sloping sides, the cross-section will become a trapezoid, and it may be theoretically shown that the best of all possible trapezoids is the half of a regular hexagon.

If we describe a semicircle with a radius r equal to the depth of the channel, we can circumscribe about that semicircle a number of rectilinear figures, each of which will form the lower half of a regular polygon having an even number of sides; and if the top water-line is made to coincide with the horizontal diameter in each case, we shall have a series of cross-sections which may be advantageously employed for an open channel. Thus, in Figs. 20, 21, 22, and 23, we have the half-square, the semi-hexagon, the semi-octagon, and the semicircle: and for each of these figures it is easy to calculate the sectional area and the hydraulic radius $R = \frac{a}{p}$.

Using b to denote the side of the polygon, and writing out the perimeter p and the sectional area a in terms of b and r, we obtain quite simply the values that are appended to each figure, and it will be noticed that in every case the hydraulic radius R, or hydraulic mean depth, is exactly one-half of the radius r of the inscribed semicircle.

The depth of the channel is always equal to r, and the top

width is equal to $2r$ in every case except the semi-hexagon, where it is $W = 2r \tan 30^\circ = 4R \tan 30^\circ$.

In calculating the discharge of such channels or in calculating the dimensions which are requisite for any given discharge, it will be convenient to express the area a in terms of R^2, making $a = CR^2$, where C is a multiple depending upon the form. The value of C is appended to each figure.

Half-square.

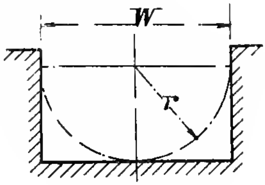
Perimeter $p = 4r$; Area $a = 2r^2$

$$\text{Hydraulic radius } R = \frac{a}{p} = \frac{r}{2}$$

$$a = 8R^2;\ C = 8$$

$$W = 2r = 4R$$

FIG. 20



Semi-hexagon.

$$\text{Side } b = 2r \tan 30^\circ$$

$$\text{Perimeter} = 6r \tan 30^\circ$$

$$\text{Area} = 3r^2 \tan 30^\circ$$

$$\text{Hydraulic radius } R = \frac{r}{2}$$

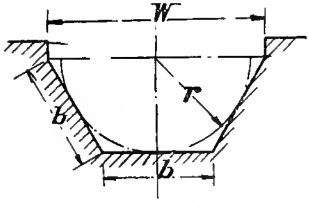
$$a = 12R^2 \tan 30^\circ$$

$$C = 12 \tan 30^\circ$$

$$W = 2r \tan 30^\circ$$

$$= 4R \sec. 30^\circ$$

FIG. 21



Semi-octagon.

$$\text{Side } b = 2r \tan 22\tfrac{1}{2}^\circ$$

$$\text{Perimeter} = 8r \tan 22\tfrac{1}{2}^\circ$$

$$\text{Area} = 4r^2 \tan 22\tfrac{1}{2}^\circ$$

$$\text{Hydraulic radius } R = \frac{r}{2}$$

$$a = 16R^2 \tan 22\tfrac{1}{2}^\circ$$

$$C = 16 \tan 22\tfrac{1}{2}^\circ$$

$$W = 2r = 4R$$

FIG. 22

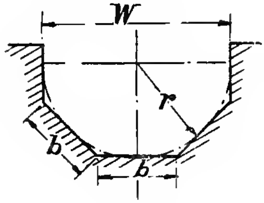
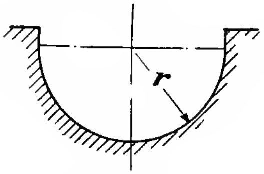


FIG. 23



Semicircle.

Perimeter $p = r\pi$; Area $a = r^2\frac{\pi}{2}$

Hydraulic radius $R = \frac{r}{2}$

$$a = 2\pi R^2;\ C = 2\pi$$
$$W = 2r = 4R$$

Art. 38. To find the Discharge from an Open Channel of Polygonal Form.—

Writing $a = CR^2$, we have in every case

$$\log. Q = \log. V + \log. a$$
$$= \log. V + 2 \log. R + \log. C$$

and, inserting the general expression for V, or,

$$\log. V = \frac{m}{n} \log. R + \frac{1}{n}(\log. s - \log. \mu),$$

we have—

$$\log. Q = \left(2 + \frac{m}{n}\right) \log. R + \frac{1}{n} (\log. s - \log. \mu) + \log. C$$

and, for either of the materials included in the Groups I., II., III., this becomes—

$$\log. Q = \tfrac{8}{3} \log. R + \frac{1}{n}(\log. s - \log. \mu) + \log. C$$

In this expression the constant log. C will have the following values, as determined in the last article :—

Form of cross-section.		
Half-square	$C = 8$;	$\log. C = 0{\cdot}903090$
Semi-hexagon	$C = 12 \tan 30°$;	$\log. C = 0{\cdot}840620$
Semi-octagon	$C = 16 \tan 22\tfrac{1}{2}°$;	$\log. C = 0{\cdot}821344$
Semicircle	$C = 2\pi$;	$\log. C = 0{\cdot}798180$

EXAMPLE.—An open channel of ordinary brickwork (somewhat rough in character) is laid to a gradient of 1 in 1000. Its cross-section below top-water is a semi-hexagon described about a circle of 5 feet radius. What will be the discharge?

For this material we have $\frac{1}{n} = \frac{5}{9}$, and $\log. \mu = \bar{4}{\cdot}050$, while

$R = \frac{r}{2} = 2{\cdot}5$ feet.

Here $s = \frac{1}{1000}$, and log. s			=	$\bar{3}\cdot000000$
Subtract log. μ			=	$\bar{4}\cdot050000$
				$0\cdot950000$
Multiply by $\frac{5}{9}$				5
				$9)4\cdot750000$
				$0\cdot527777$
R = 2·5, and log. R	=	0·397940		
Multiply by $\frac{8}{3}$		8		
		3)3·183520		
Add		1·061173		1·061173
Add also log. C			=	0·840620
Then log. Q			=	2·429570

Whence Q = 275 cubic feet per second.

In this channel the top width is, of course, greater than 10 feet. It is $W = 10 \times \sec. 30° = 11\cdot55$ feet.

Art. 39. To find the Transverse Dimensions of a Channel of Regular Polygonal Form, when the Discharge Q and the Gradient *s* are given.—For this purpose it is only necessary to transpose the equation employed in the preceding article. The logarithm of the hydraulic radius R will be given by the general expression—

$$\log. R = \frac{n}{2n + m}\left\{\log. Q + \frac{1}{n}(\log. \mu - \log. s) - \log. C\right\}$$

For any of the materials in Groups I., II., III., this becomes—

$$\log. R = \tfrac{3}{8}\left\{\log. Q + \frac{1}{n}(\log. \mu - \log. s)\right\} - \tfrac{3}{8}\log. C$$

and, in every case, the radius of the inscribed semicircle is $r = 2R$.

Example.—If an open channel, lined with ordinary brickwork, is to be constructed in the form of a half-square, and laid to the same gradient of 1 in 1000, as in the last example, what must be its width and depth in order that it may convey the same discharge of 275 cubic feet per second?

$Q = 275$, and consequently log. Q		$= 2{\cdot}429570$
Taking, as before, log. μ	$= \bar{4}{\cdot}050000$	
Subtract log. s	$= \bar{3}{\cdot}000000$	
giving log. μ − log. s	$= \bar{1}{\cdot}050000$	
whose negative value is	$-0{\cdot}950000$	
Multiply by $\frac{1}{n}$ or $\frac{5}{9}$	5	
	$9)-4{\cdot}750000$	
	$-0{\cdot}527777$	
Add this negative quantity, or subtract		$0{\cdot}527777$
giving log. $Q + \frac{1}{n}$(log. μ − log. s)		$= 1{\cdot}901793$
Multiply by $\frac{3}{8}$		3
		$8)5{\cdot}705379$
		$0{\cdot}713172$
For the half square log. $C = 0{\cdot}903090$		
subtract $\frac{3}{8}$ log. C		$= 0{\cdot}338678$
and we have log. R		$= 0{\cdot}374494$

Whence $R = 2{\cdot}369$.

The depth must, therefore, be $2R = 4{\cdot}738$ feet

and the width must be $4R = 9{\cdot}476$ feet

If we adopt the same value of μ for all forms of cross-section, it will be a simple matter to determine the requisite dimensions for any alternative polygonal form. For the given discharge $Q = 275$ and the given gradient, we shall have in each case—

$$\tfrac{3}{8}\left\{\text{log. } Q + \frac{1}{n}(\text{log. } \mu - \text{log. } s)\right\} = 0{\cdot}713172$$

and it will only be necessary to subtract the proper value of $\frac{3}{8}$ log. C.

Thus, for the semi-hexagon log. $C = 0{\cdot}840620$, and subtracting $\frac{3}{8}$ log. C, we have—

$$\text{log. } R = 0{\cdot}713172 - 0{\cdot}315232 = 0{\cdot}397940$$

whence $R = 2{\cdot}5$, as in the last example.

For the semi-octagon log. $C = 0{\cdot}821344$, and, subtracting three-eighths of this—

$$\text{log. } R = 0{\cdot}713172 - 0{\cdot}308008 = 0{\cdot}405164$$

whence $R = 2{\cdot}542$

So that the depth is $2R = 5{\cdot}084$ feet

and the width is $4R = 10{\cdot}168$ feet

For the semicircle log. C = 0·798180, and, treating this in the same way, we should get theoretically—

$$\text{Log. R} = 0{\cdot}713172 - 0{\cdot}299317 = 0{\cdot}413855$$

whence R = 593, and the semicircle would have a radius r = 2R = 5·186 feet, or a diameter of 10·372, or, say, 10 feet $4\frac{1}{2}$ inches.

It is worth noticing, perhaps, that, among these four designs for the conveyance of the same quantity of water upon the same gradient, the depth of channel, the sectional area, and the width of wetted perimeter, come out at the following figures:—

	Depth.	Area.	Perimeter.
Half-square	2·369	44·88	18·95
Semi-hexagon	2·500	43·30	17·32
Semi-octagon	2·542	42·82	16·85
Semicircle	2·593	42·26	16·30

In the case of the semicircle, it would, no doubt, be more correct to take a somewhat lower value for the constant μ; and the advantage of the circular form would then appear to be more conspicuous. Thus, if we take log. $\mu = \bar{5}{\cdot}975$, the calculated radius for the semicircle would be reduced to 2·50 feet, and with that radius the sectional area would be only 39·32 square feet, and the perimeter would be 15·72 feet.

Art. 40. Other Forms of Cross Section.—In practice the regular polygonal form would often be used for small aqueducts, but not often for flood-water channels of large dimensions. The flat floor will very frequently be replaced by a dished invert, and, when the side walls have to be treated as retaining walls, their design will often be dictated by considerations of stability. For similar reasons, it will often be advantageous, in the design of very large works, to adopt a width of channel which is much greater than twice the depth, and in other respects the form of the design will be influenced by local conditions.

When the proportions are thus arbitrarily adjusted, it is, of course, impossible to deduce any rule for the direct calculation of the transverse dimensions. The section must be provisionally outlined upon paper, and then it will be a simple matter to measure the area a, the perimeter p, and the hydraulic radius R, and to calculate the discharge Q by the methods which have already been described.

Thus, by a few tentative readjustments the required section of channel will presently be found; and the same tentative method

will generally be employed in preference to any other when the problem consists in finding the flood-level to which the water will rise in a channel of known section, when the flood-discharge reaches a given magnitude. Thus, for any large and important work a table will generally be calculated, giving the discharge of the channel for each successive foot of elevation of the water-level, and such a table will furnish all the information that is required.

Art. 41. Allowance for Probable Error.—It is sufficiently obvious already that the calculations which have been discussed in this chapter can never be made with anything like absolute accuracy, whatever may be the nature of the formula employed. For the question is mainly concerned with a certain kind of frictional resistance, and in such problems everything depends upon the actual character of the surfaces.

If we could assume for each specified class of work a uniformity of surface, the probable errors *of the formula* might perhaps be judged from the columns of Tables 4 to 11 by comparing the calculated with the observed velocity. *Those* errors are not large—very generally less than $1\frac{1}{2}$ per cent., and seldom greater than 3 or 4 per cent.; but this agreement is due to the fact that in each series the surface has remained the same, while the other conditions have been varied, and it proves that, for a given kind of surface, the formula is good enough under all ordinary conditions.

But if the constants that were used for the very smooth brickwork of Class 4 had been applied to the rougher brickwork of Class 8, the divergence would have been much greater, amounting to 20 or 25 per cent., as shown in the calculated discharges of Table 13.

In every class of material a similar divergence is frequently to be found, and would be abundantly illustrated if the formula were applied to *all* the gaugings that have been recorded at various times and places.

The constants have been determined from the experimental results obtained in certain examples which have been selected from amongst others because they are well authenticated and because they have yielded the most consistent and regular results; but they are precisely the cases which generally yield the highest discharges. As compared with these we shall very often find gaugings showing a discharge which is not more than 90 per cent., several in which it ranges from 80 to 70 per cent., and a few in which it is yet lower.

It is reasonable to believe that where such lower figures have

been recorded the flow may have been impeded by sediment or some other obstruction, or merely by a deterioration of the internal surface; but this last contingency will not be dismissed from the practical calculations of the engineer. To be on the safe side he will not wish to underestimate the discharge which he has to deal with, nor to overestimate the discharging capacity of the conduit which he is designing. For the first of these purposes the formula may fairly be taken as giving the maximum; but to provide against a contingent inferiority in the character of the surface, it would always be reasonable to make use of some multiple or factor of safety applied to the calculated discharge. Thus the diameter of a proposed conduit in any material might rationally be determined by using the formula with constants appropriate to that material, if the required discharge were multiplied by such a factor. In no case would the factor be less than 1·10, and it should probably be greater if the material is one which presents wide variations in the usual roughness of its surface. It would have to be very much greater if the factor were intended to cover the contingency of any future growth of vegetation, or accretion of slimy deposit, or the deep pitting of an iron pipe by rust.

Art. 42. The Coefficient in the Old Formula.—Everybody knows that if hydraulic calculations are to yield results which shall bear any reasonable resemblance to the truth, the old formula must either be abandoned or its coefficient must be taken at some carefully selected value which varies with all the varying conditions. In the expression $V = c\sqrt{Rs}$, the true value of the coefficient c will depend upon three or four circumstances apart from the temperature of the water:—

(1) It will depend upon the roughness of the material which lines the conduit.

(2) In conduits of the same material it will depend upon the diameter of the pipe or the hydraulic radius R of the channel.

(3) In each case it will, again, depend upon the gradient s.

(4) Although it will not differ greatly in different forms of cross-section, it will certainly have a somewhat greater value in circular and curvilinear sections than in channels of rectangular or polygonal form.

But with all this ambiguity the old formula has been retained in preference to any other, and it is probable that engineers will continue to use it in some of their calculations on account of its great convenience for many practical purposes. For such

calculations the engineer will endeavour to select a suitable value for the coefficient by reference to some recorded experiment in which the governing conditions have resembled those of his own particular case.

But so far as the influence of those conditions can be gathered from recorded experiments, their separate effects have been analyzed in the preceding articles, and the following tables give the values of the coefficient c as computed in accordance with the formulæ deduced in those articles.

The values here given will be found to agree very well with the observed facts so far as the recorded observations have extended; but it must evidently be somewhat hazardous to fill in the figures at the extreme corners of the tables, for here we are approaching conditions which lie far *beyond* the range of any actual gaugings.

TABLES OF COMPUTED VALUES FOR THE COEFFICIENT c IN THE USUAL FORMULA $V = c\sqrt{Rs}$.

TABLE 14.—COEFFICIENT FOR CIRCULAR PIPES OF CLASS 1, WITH A SMOOTH GLAZED LINING.

GRADIENT.	DIAMETER.								
	1 in.	3 in.	6 in.	1 foot	2 feet	3 feet	4 feet	5 feet	6 feet
1 in 5	106	127	143	160	180				
„ 10	101	121	136	153	171	183			
„ 20	96	115	129	145	163	174	183		
„ 40	92	110	123	138	155	166	174	181	
„ 60	89	107	120	134	151	161	169	176	181
„ 100	86	103	115	130	145	155	163	169	174
„ 200	81	98	110	123	138	148	155	161	166
„ 400	77	93	104	117	132	141	148	153	158
„ 600	75	90	101	114	128	137	143	149	153
„ 1000	73	87	98	110	123	132	138	144	148
„ 2000	69	83	93	104	117	125	132	136	142
„ 4000	65	79	88	98	111	118	125	128	135

NOTE.—These values are not reached in pipes of bare metal, with ordinary socket joints, nor in riveted pipes. For a semi-circular channel, with very smooth lining of neat cement, the coefficient is higher by about 10 per cent.; but with a rendering of cement and sand it falls 4 per cent. below these figures.

TABLE 15.—COEFFICIENT FOR CLEAN PIPES OF BARE METAL, RIVETED WROUGHT IRON, AND CAST IRON SOCKET PIPES. CLASS 6.

GRADIENT.	DIAMETER.								
	1 in.	3 in.	6 in.	1 foot	2 feet	3 feet	4 feet	5 feet	6 feet
1 in 5	93	111	125	141	158	169			
„ 10	89	107	120	134	151	161	169		
„ 20	85	102	114	128	144	154	162	168	
„ 40	81	97	109	123	138	148	155	161	166
„ 60	79	95	106	120	134	144	151	156	161
„ 100	76	92	103	116	130	139	146	151	156
„ 200	73	88	99	111	124	133	139	145	149
„ 400	70	84	94	106	119	127	133	138	142
„ 600	68	82	92	103	116	124	130	135	139
„ 1000	66	79	89	100	112	120	125	130	134
„ 2000	63	75	85	95	107	114	120	125	128
„ 4000	60	72	81	91	102	109	115	119	123

NOTE.—The coefficient will fall below these values whenever the metal surface is deteriorated by rust or sediment.

TABLE 16.—COEFFICIENT FOR CHANNELS OF RECTANGULAR OR POLYGONAL FORM, IN VERY SMOOTH ASHLAR MASONRY, OR IN HARD SMOOTH BRICK WITH CAREFULLY POINTED JOINTS. CLASS 5.

GRADIENT.	HYDRAULIC MEAN RADIUS R.									
	0·25	0·50	0·75	1·0	1·25	1·50	2·0	2·5	3·0	4·0
1 in 10	137	153	164	172	178	184				
„ 20	131	147	157	165	171	177	185			
„ 40	125	140	150	157	163	168	176	183		
„ 60	121	136	146	153	158	163	171	178	183	
„ 100	117	131	140	147	153	157	165	171	177	185
„ 200	111	125	133	140	145	150	157	163	168	176
„ 400	106	119	127	133	138	143	150	155	160	168
„ 600	103	115	123	129	134	138	145	151	155	163
„ 1000	99	111	119	125	130	134	140	145	150	157
„ 2000	94	106	113	119	123	127	133	138	143	150
„ 4000	89	101	108	114	117	120	126	131	136	143

NOTE.—In culverts of oval or circular form the coefficient appears to be about 8 per cent. higher; but in every case it will depend upon the character of the workmanship.

TABLE 17.—COEFFICIENT FOR CHANNELS OF RECTANGULAR FORM IN ORDINARY BRICKWORK OR ASHLAR MASONRY. CLASSES 8A AND 9.

GRADIENT.	HYDRAULIC MEAN RADIUS R.									
	0·25	0·50	0·75	1·0	1·25	1·50	2·0	2·5	3·0	4·0
1 in 10	109	123	131	138	143	147	154			
„ 20	105	118	126	132	137	142	149	154		
„ 40	101	114	121	127	132	136	143	148	153	
„ 60	99	111	119	125	129	133	140	145	150	157
„ 100	96	108	115	121	126	130	136	141	145	152
„ 200	93	104	111	117	121	125	131	136	140	147
„ 400	89	100	107	112	116	120	126	131	135	141
„ 600	87	97	104	110	114	117	123	128	132	138
„ 1000	84	94	102	107	111	114	120	124	128	134
„ 2000	81	91	98	103	106	110	115	119	123	129
„ 4000	78	88	94	99	101	106	111	114	118	124

NOTE.—Here also it appears that for circular brick culverts 8 or 10 per cent. may be added to the figures; but in every case the coefficient will be greatly reduced by the presence of any moss or vegetation upon the walls.

TABLE 18.—COEFFICIENT FOR RECTANGULAR CHANNELS LINED WITH TIMBER PLANKING. CLASS 7.

GRADIENT.	HYDRAULIC MEAN RADIUS R.									
	0·25	0·50	0·75	1·0	1·25	1·50	2·0	2·5	3·0	4·0
1 in 10	118	132	142	149	154	159	167			
„ 20	113	127	136	143	148	153	160	167		
„ 40	109	123	131	138	143	147	154	160	165	
„ 60	107	120	128	135	140	144	151	157	162	169
„ 100	104	117	125	131	136	140	147	152	157	164
„ 200	100	112	120	126	131	135	141	147	151	159
„ 400	96	108	115	121	126	130	136	141	145	153
„ 600	94	105	113	118	123	127	133	138	142	149
„ 1000	91	103	110	115	119	123	129	134	138	145
„ 2000	88	99	106	111	115	118	124	129	133	140
„ 4000	85	95	102	107	111	113	119	124	128	135

TABLE 19.—COEFFICIENT FOR CHANNELS IN HAMMER-DRESSED MASONRY. CLASS 12.

GRADIENT.	HYDRAULIC MEAN RADIUS R.									
	0·25	0·50	0·75	1·0	1·25	1·50	2·0	2·5	3·0	4·0
1 in 10	70	82	89	95	99	103	110	115	120	127
„ 20	69	81	88	94	99	102	109	114	119	126
„ 40	69	81	87	93	98	102	108	113	118	125
„ 60	68	80	87	93	98	101	108	113	118	125
„ 100	68	80	86	92	97	101	107	112	117	124
„ 200	68	79	86	92	96	100	107	112	117	123
„ 400	67	79	85	91	95	99	106	111	116	122
„ 600	67	78	85	91	95	99	105	111	115	122
„ 1000	67	78	84	90	94	98	104	110	114	121
„ 2000	66	77	84	90	94	98	104	109	113	120
„ 4000	66	76	83	89	93	97	103	108	112	120

NOTE.—The coefficient is probably still lower in the case of rock-faced masonry presenting rough protuberances. It is lower than the figures above given, in many examples of rough rubble masonry.

CHAPTER VI.

INLETS AND OUTLETS.

Art. 43.—In the preceding chapters we have been dealing with those frictional resistances which govern the flow of water as it travels along a conduit of uniform section with a constant velocity, but we now have to look at the changes of velocity which take place at the first entrance of the water through the inlet and its final delivery through the outlet; for these changes of velocity will involve certain transformations of energy, and must be accompanied by certain changes of head or pressure.

At the first start, the work of acceleration will involve a loss of head which must at least be equivalent to the acquired kinetic energy; and when the water comes to rest at the end of its journey this kinetic energy will be given up, and must either be reconverted into head or dissipated in the performance of frictional work, or in the production of eddies.

These several changes and concurrent effects will depend, however, upon the form of the adjutage which constitutes the inlet or the outlet of the pipe or reservoir. In some cases the water may find its ingress or its egress through an orifice in a thin wall, in many cases it will enter and leave the pipe through its plain cylindrical ends, while sometimes the pipe or culvert may have a converging inlet of conical or conoidal form, and possibly a diverging outlet of similar shape.

Experiments have been made with orifices and adjutages of these several shapes applied as discharging outlets from a still-water reservoir, and the observed results have sometimes been stated in such a form that they almost look like a paradox.

The head h has been measured from the horizontal axis of the orifice to the surface of the water, and a certain velocity equal to $\sqrt{2gh}$ has been taken as the velocity theoretically due to that head. Then multiplying this "theoretical" velocity by the sectional area a_0 of the outlet, measured at some particular point,

the product $a_0\sqrt{2gh}$ has been taken to represent a "theoretical discharge." The actual discharge Q is found to differ greatly from this calculated quantity, and such a difference might rationally be expected; but for some reason it has been deemed convenient to express the actual discharge in terms of the quantity $a_0\sqrt{gh}$ with the aid of a "coefficient of discharge," C.

Thus in one case it is found that the actual discharge is not greater than $Q = 0{\cdot}5\ a_0\sqrt{2gh}$, while outlets of other forms give a discharge two or three or four times as great, so that the coefficient C has widely differing values in the different forms of adjutage.

But these values of the coefficient do not convey any intelligible information; they present us with a comparison between figures which are essentially incomparable, and therefore they seem to imply a contradiction which does not really exist. The wide differences which they exhibit, are differences which might rationally be expected, and do not in any way prove that experiment is at variance with theory; for in all cases the facts are consistent with known dynamical principles, and the coefficient might almost be calculated beforehand.

It will be useful to examine some of the cases which most frequently occur in engineering practice, and to notice how the expended energy is devoted to the performance of the work of acceleration, or of frictional work, or is wasted in any other way.

Art. 44. The Work of Acceleration.—One pound of water moving at any given velocity v possesses the kinetic energy $\frac{v^2}{2g}$, and if $Q\gamma$ is the weight in pounds of any measured quantity of water moving at that velocity, its kinetic energy will be $Q\gamma\frac{v^2}{2g}$. If the velocity has been acquired by the action of gravity upon the same mass of water while it descends through a certain height h, the energy expended in that fall will be simply $Q\gamma h$; and if the whole of this expended energy is devoted to the work of acceleration, it will have been wholly converted into kinetic energy, so that

$$Q\gamma h = Q\gamma\,\frac{v^2}{2g},$$

$$\text{or } h = \frac{v^2}{2g} \qquad (2)$$

and in such a case the velocity due to any given loss of head h would be $v = \sqrt{2gh}$ . . (2a)

But here we must remember that the expended energy is not always to be measured by the *visible* height of fall. The *effective* loss of head may often have to be measured by a fall of *pressure* which may be expressed either in inches of mercury, or in pounds per square inch, or in feet of an equivalent head of water.

Thus, for example, when a jet of water is turned into the condenser of a steam-engine, the velocity of the jet is not to be calculated from the mere height of the water-surface above the orifice. If the visible head of water measures 1 foot, the velocity due to that head alone would be about 8 feet per second; but if the vacuum-gauge registers at the same time 26 inches of mercury, which would be equivalent to about 30 feet of water, the *effective* loss of head would amount to 31 feet, and the corresponding velocity of the jet would be about $8\sqrt{31} = 44$ feet per second.

To a somewhat smaller extent, the effect of such negative pressures is quite clearly to be seen in some forms of pipe-inlet; and it goes far to explain some of the paradoxical results which were referred to in the last article.

In making use of formula (2, 2_a), therefore, we must remember that the visible height of fall will only be applicable when it represents the effective loss of head, and is consequently a true measure of the expended energy.

Art. 45. Wasted Energy.—When the acceleration above spoken of is taking place at the inlet of any conduit, we can hardly expect that the whole of the expended energy will reappear in the kinetic energy of the current, for during the process of acceleration it is almost certain that some little work will be done against frictional resistances at the entrance, and it is possible that work may be wasted in other ways.

To produce a given velocity v it would require therefore an effective loss of head h_1 at the inlet of the pipe, somewhat *greater* than $\frac{v^2}{2g}$; and if we measure the work actually lost in this way by the additional loss of head z, we may write generally—

$$h_1 = \frac{v^2}{2g} + z$$

It has been commonly assumed that the loss z is itself proportional to v^2, and on this assumption we might say that $z = \zeta_1 \frac{v^2}{2g}$, where ζ_1 is a coefficient to be determined by experiment.

The formula then becomes—

$$h_1 = \frac{v^2}{2g}(1 + \zeta_1) \qquad (2c)$$

as in the writings of Weisbach and other hydraulicians.

Another way of stating the same fact is to say that the actual velocity v due to any given loss of head h_1 at the inlet, will be somewhat *smaller* than the quantity $\sqrt{2gh_1}$, and may be expressed in terms of that quantity by using a "coefficient of velocity" C_v, and thus writing—

$$v = C_v\sqrt{2gh_1} \quad . \qquad (2b)$$

In most cases C_v is only slightly less than unity.

Art. 46. The Contracted Vein.—In making any theoretical calculation of the discharge Q by multiplying together the velocity and the sectional area of the current, it is sufficiently obvious that we must measure the area a at the same section where the velocity v is to be found.

When water is allowed to flow out of a reservoir through a sharp-edged circular orifice, it always issues in the form of a converging conoidal jet, which comes to a truly cylindrical form at a short distance in front of the orifice, where the sectional area a is considerably smaller than the area a_0 of the orifice itself. In traversing this short distance the sectional area is continually diminishing, which means, of course, that the velocity is continually increasing; and therefore the work of acceleration is not completed until the stream reaches the point where the minimum section a is found, and where the jet becomes cylindrical in form.

Hence, it follows quite clearly that, if the velocity v is calculated by either of the formulæ ($2a$) or ($2b$), given in the last two articles, it can only be taken as representing the velocity of the current at the small section a, where the acceleration is completed; and we cannot, on rational grounds, expect the discharge to be greater than $Q = a\sqrt{2gh}$, a quantity considerably smaller than the so-called "theoretical discharge" $a_0\sqrt{2gh}$, because the contracted area a is so much smaller than the area of the orifice a_0.

In dealing with practical engineering questions, or in recording the results of an experiment, it is much easier to define the area a_0 of the actual orifice than to determine the exact area a of the contracted stream, and, for that reason, it may be convenient to employ the quantity $a_0\sqrt{2gh}$ as a standard. But whenever the

diameter of the contracted jet can be accurately measured or estimated, the ratio $\frac{a}{a_0}$ may be usefully expressed by a "coefficient of contraction," making $C_a = \frac{a}{a_0}$.

The true discharge will then be expressed by $Q = C_a v a_0$, in which v is the maximum velocity of the current at the small section. At the same time v may be determined from the loss of head h_1, by the formula $C_v\sqrt{2gh_1}$, and the discharge in terms of the known dimensions h_1 and a_0 will then be—

$$Q = C_a C_v a_0 \sqrt{2gh_1}$$

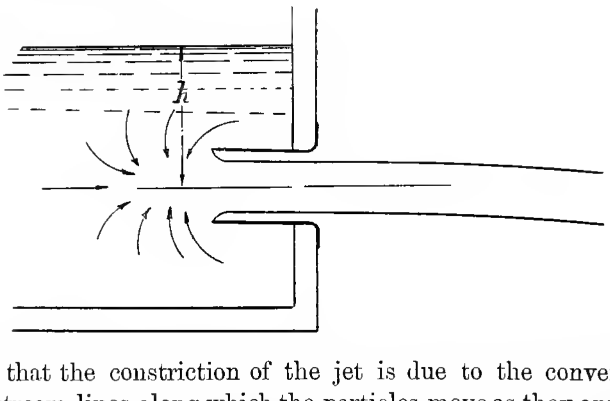
or, more shortly, $Q = Ca_0\sqrt{2gh_1}$

where C is the so-called coefficient of discharge, and equal to the product C_aC_v.

Of these three coefficients, it is evident that any one of them can be calculated if the other two are known.

With regard to the coefficient of contraction C_a, it is well-

FIG. 24



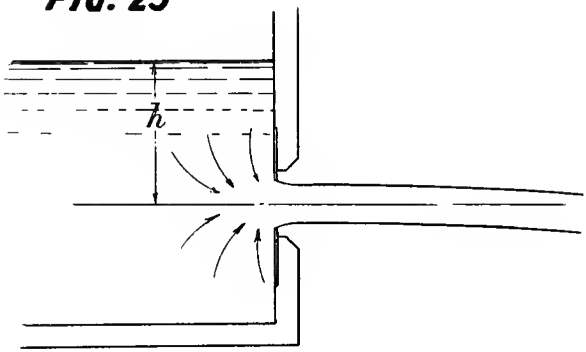
known that the constriction of the jet is due to the convergence of the stream-lines along which the particles move as they approach the opening; and the constriction is sharpest when the stream-lines have the fullest liberty of converging towards the orifice from *all* directions. A thin tube projecting a short distance inside the reservoir, as sketched in Fig. 24, would best fulfil this condition; and the measured discharge from such an outlet would give a coefficient of discharge C not much greater than 0·50, or perhaps, 0·53. The stream-lines, converging from all directions,

would so sharply contract the issuing jet that the water would not touch the sides of the tube, but would form a central cylinder, whose diameter would be not much greater than seven-tenths of the diameter of the tube, its area a being not much more than one-half of the area a_0.

If a circular orifice of the same diameter were made in a *vertical plate*, flush with the inner surface of the reservoir wall, the discharge would be considerably greater for the same head, the coefficient C being then about 0·60, or a little higher. For, in this case, the convergence of the stream-lines would be limited by the plane surface of the plate, as illustrated in Fig. 25. Here the stream-lines, at any radial section, can only converge from points which extend round an arc of 90°, while in Fig. 24 the convergence may extend round 180°.

Between these two cases, it would be easy to introduce any number of intermediate ones. Thus, if the metal plate were

FIG. 25



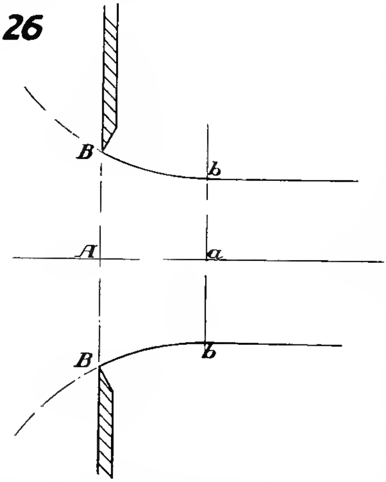
dished inwards in the form of a flat cone, the arc of convergence would be increased to some angle between 90° and 180°, according to the taper of the cone; and the diameter of the contracted stream would be affected by each change in the angle, while the coefficient of discharge would be found to have intermediate values, ranging between 0·60 and 0·50. Conversely the arc of convergence might be reduced to something less than 90° by simply turning the dished plate in the opposite direction with the apex of the cone outwards; and if a_0 is always understood to be the area of the orifice at the apex of the cone, the coefficient of discharge will increase from 0·60 to larger values, depending in each case upon

the taper of the cone, until a coefficient approaching to unity is obtained, as it may be with a suitable form of conoidal outlet.

Amongst these various forms, the aperture in the flat plate will possess a special interest in connection with some practical applications. The longitudinal section drawn in Fig. 26 may be taken as representing pretty nearly the form of the contracted vein as it issues in free air from a circular orifice, which has been carefully made in a thin plate, and countersunk on the outer side so as to produce a sharp cutting edge.

So long as the water stands in the reservoir at a good height above the orifice, the proportions of the contracted vein appear to be nearly the same for all diameters of orifice, and show no considerable change with any further increase of head.

FIG. 26



From the vertical section bb onwards, the water forms a true and beautifully polished cylinder, showing no sign of scratch or ripple or opaqueness; and from the orifice BB to this section the stream converges in a conoidal form with a curved section Bb.

According to the measurements of Michelotti and D'Aubuisson, the length Aa is $l_0 = 0{\cdot}498\text{D}$, where D is the diameter BB of the orifice; while the contracted diameter bb is $d = 0{\cdot}787\text{D}$.

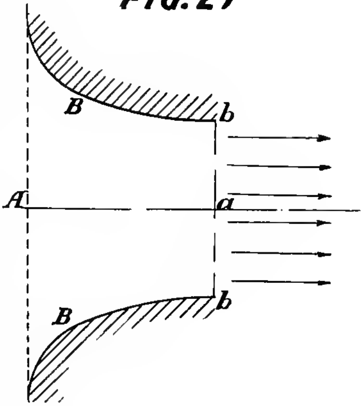
These figures would, of course, give the coefficient of contraction $\text{C}_a = \dfrac{a}{a_0} = 0{\cdot}787^2 = 0{\cdot}619$. It must be remarked, however, that an exact measurement of the diameter d in any experiment is a somewhat difficult matter. Some observers have found $d = 0{\cdot}79\text{D}$, which gives $\text{C}_a = 0{\cdot}624$; while Weisbach measures the diameter at $d = 0{\cdot}80\text{D}$, which makes $\text{C} = 0{\cdot}640$.

A slight rounding of the sharp edge, or a very small interference with the convergence of the stream-lines, would be enough to account for these little divergencies of measurement. Thus, if the

floor of the reservoir in Fig. 25 were raised to within a short distance from the lower edge of the orifice, the contraction would certainly be less, and the coefficient C by so much greater, owing to the shutting off of convergent stream-lines from below; and it is obvious that a similar effect would be produced in the upper region if the water-level in the reservoir were allowed to fall so low as to leave only a scanty depth above the top of the orifice. The high coefficient of discharge, which has commonly been observed at very low heads, is undoubtedly due to such an increase in the coefficient of contraction; and shows, indeed, that, with very low heads, C_a must be greater than 0·64, for it is not possible that C_a can be less than the coefficient of discharge C, whose value will presently be considered.

Art. 47. Conoidal Inlets and Outlets.—To obtain the greatest possible discharge through a short pipe or culvert of given diameter, with a given head of water, it will be advantageous in many cases to construct the inlet in a conoidal form following pretty nearly the lines of the contracted vein. The proportions adopted for such inlets have varied in different examples, and have depended partly upon the magnitude of the work; but the section drawn in Fig. 27 will serve to illustrate the typical form.

FIG. 27



At the section *bb*, the trumpet mouth dies into the cylindrical surface of the barrel; and the curve *Bb* follows approximately the line of Fig. 26; but beyond *B* the curve is carried on with a sharper radius until it dies into the plane wall surface, at right angles to the axis of the conduit.

A short mouthpiece of the same form, terminating in an open nozzle at *bb*, has often been used as an outlet from a reservoir, discharging its jet into free air at the nozzle *bb*.

In either case the converging stream-lines follow naturally along the curve *Bb*, or very close to it, and issue at the section *bb* in lines parallel to the axis, so that the inlet-stream enters the cylindrical barrel without any further contraction, while the conoidal outlet in the same manner delivers its jet at *bb* into

free air in the form of a true cylinder, whose diameter and sectional area are precisely the same as those of the orifice at bb, and can be accurately measured.

Hence the experiments that have been made with the conoidal outlet possess a definiteness which was lacking in the case of the contracted vein. The polished metal mouthpiece becomes *the gauge* for measuring the area a, the coefficient of contraction C_a is exactly unity, and the coefficient of discharge C is the same thing as the coefficient of velocity C_v. The work that is wasted in friction or otherwise can therefore be accurately measured, and the recorded experiments of Weisbach, Borda, Michelotti, and Eytelwein, show that the loss of head due to this wasted work is often not more than 3 or 4 per cent. of the dynamic head $\frac{v^2}{2g}$.

But in a properly formed mouthpiece this loss z must be almost wholly due to the frictional resistance of the conoidal surface; and the experiments show, as might be expected, that it is not really proportional to v^2, but more nearly proportional to $v^{1\cdot75}$, as it would be in a cylindrical pipe according to the formula given in Chapter IV.

In fact, if we calculate the frictional loss of head h_2 upon a cylindrical pipe whose diameter is $d = bb$, and whose length is $l_o = Aa$ (the actual length of the mouthpiece), it will be very nearly sufficient to account for the difference between the observed velocity v in Weisbach's experiments and the theoretical velocity $\sqrt{2gh}$.

In these experiments the mouthpiece was made in brightly polished gunmetal, for which surface we may take the constants given for Class 1. The diameter at the nozzle was 1 centimetre, or 0·0328 feet, so that R = 0·0082, while the length of the mouthpiece appears to have been nearly $2\frac{1}{4}$ times the diameter, or say $l_o = 0{\cdot}074$. The discharge was measured under widely varying heads from 2 centimetres to 103 metres, and in the following table the observed coefficient of discharge is given for each experiment, along with the value that might be calculated for that coefficient if the frictional loss of head is reckoned by the formula of Chapter IV.

A similar result will be obtained if the experiments which have been recorded by Michelotti are examined in the same way. That is to say, the actual velocity of the issuing stream was very nearly the velocity which might be calculated from the head $h - z$,

which is obtained by subtracting from the actual head h the loss of head due to pipe-friction in a length l_0 equal to the length of the mouthpiece. In these experiments the actual length of mouthpiece appears to have been again very nearly $2\frac{1}{4}d$, so far as can be seen from the half-scale section reproduced in Weisbach's "Mechanik."

TABLE 20.—DISCHARGE OF CONOIDAL OUTLET.

Experiment.	Number.				
	251	252	253	254	255
Head in feet, $h =$	0·0656	1·6400	11·485	55·780	337·900
Observed velocity, v in feet per sec.	1·971	9·9380	26·510	59·560	146·600
Frictional loss of head on the length l_0, due to observed velocity, or $z = l_0\mu\frac{v^n}{R^m}$	0·0050	0·0838	0·469	1·923	9·300
Calculated effective head, or $h - z$	0·0606	1·5560	11·016	53·857	328·600
Calculated velocity at nozzle, or $\sqrt{2g(h-z)}$	1·9750	10·0100	26·630	58·900	147·500
Calculated coefficient of velocity C''	0·9610	0·9730	0·979	0·983	0·987
Observed „ „	0·9590	0·9670	0.975	0·994	0·994

According to the observed values of the coefficient $C_{''}$, as quoted in the above table, the term ζ_1 in formula (2c) would have the value $\left(\frac{1}{C_{''}}\right)^2 - 1$ ranging from $1\frac{1}{4}$ to $8\frac{3}{4}$ per cent.: but Weisbach takes the average value $C'' = 0·975$, which gives $\zeta_1 = 0·052$ nearly, so that he writes—

$$h_1 = 1·05\frac{v^2}{2g}; \text{ or, } z = 0·05\frac{v^2}{2g}$$

and this value of z is nearly equivalent to the frictional loss of head which, according to the old formula, would occur in a pipe whose diameter is d and its length $2d$. It would appear, however, that a closer estimate might be made by reckoning z as a frictional loss of head proportional to $v^{1·75}$, as it is very nearly in this example. At the conoidal inlet of a masonry culvert the frictional loss may be greater than in a conoidal inlet of polished gunmetal; but in calculating the total loss of head for such a culvert we shall hardly go wrong if we take the dynamic head $h_1 = \frac{V^2}{2g}$, and

add to it the frictional loss of head h_2 reckoned upon the entire length of the culvert including the inlet—provided that the length of inlet is not less than $2d$ or $2\frac{1}{4}d$.

Art. 48. Outlet through an Orifice in a Thin Vertical Plate.—When the water is discharged through a sharp-edged orifice into free air, the visible contraction of the jet outside the plate can be approximately measured, and its dimensions have already been referred to in Art. 43, and illustrated in Fig. 26.

So long as the stream contracts in area the velocity continues to increase, and the acceleration goes on until the section *bb* is reached, where the work of acceleration is completed. It is only at this point, therefore, that we could expect to find the velocity due to the fall h; and even here the velocity v would not be quite equal to $\sqrt{2gh}$ if any portion of the expended energy had been wasted on the way.

In passing through the air from B to b (Fig. 26), the smooth surface of the conoidal jet would experience so little resistance that we may leave it out of account, and there remains only such frictional work as may be due to the shearing action of the sharp edge and the passage of the converging water-streams through the water of the reservoir behind the plate. Hence we may expect to find that the wasted head z is really very small, and that the ratio $\frac{v}{\sqrt{2gh}}$ or the coefficient of velocity C_v measured at the section *bb* is almost equal to unity; or, in other words, that the measured coefficient of discharge C is only a little less than the measured coefficient of contraction C_a.

Taking the circular form of orifice, it will be remembered that the diameter of the contracted vein as measured by Michelotti, Weisbach, and other observers, gives a coefficient of contraction C_a from 0·619 to 0·640.

Turning, then, to the numerous experiments that have been made to determine the *discharge* from similar orifices, we find that C is not much smaller than C_a. In the experiments of Bossut and Eytelwein, with heads varying from 2 to 12 feet, the coefficient of discharge varies from 0·619 to 0·617; and, within the same range of head the measurements of Michelotti give values from 0·619 to 0·607; while Weisbach, working with heads less than 2 feet, found some higher values, the coefficient ranging from 0·637 to 0·607, and reaching the highest figures when the head is lowest.

Comparing these figures for the two coefficients, it is evident that the discharge is not curtailed to any considerable extent by frictional losses of any kind, but depends almost wholly upon the contracted sectional area a.

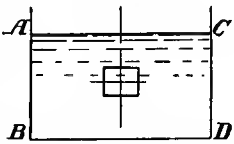
Weisbach takes 0·62 as an *average* value of the coefficient of discharge, and 0·64 for the contraction, so that the coefficient of velocity at the section bb is—

$$C_v = \frac{C}{C} = \frac{0{\cdot}62}{0{\cdot}64} = 0{\cdot}97$$

This would make $\zeta_1 = 0{\cdot}062$; and the head wasted in frictional work, or dissipated in eddies, would be $z = 0{\cdot}062\frac{v^2}{2g}$,—a slightly greater loss than was found in the conoidal mouthpiece of polished gunmetal.

For practical purposes we are chiefly concerned in getting a reliable estimate of the discharge from an orifice of known dimensions, and it is worth noticing that the coefficient is never less than 0·59, and very seldom less than 0·60. This appears to be very nearly its value for the greatest heads, whether the opening be large or small, and whether it be circular, square, or rectangular in form; but with lower heads the coefficient will rise to some higher value depending upon the size of the orifice. The rise is not great in the case of large openings, but in small ones the coefficient may reach 0·65 or 0·66, such large values being found only when the water-level sinks to within a short distance from the upper edge of the orifice, cutting off the natural convergence of the stream-lines from the upper side.[1] This is sufficiently illustrated by the results quoted in the next ensuing article, in Table 21, where the coefficient shows very little change until we come to Experiments Nos. 265 and 266.

FIG. 28



A similar increase is observed when *either* of the boundaries AB, BD, or DC, is brought near to the edge of the orifice in Fig. 28; and in each case these observed changes

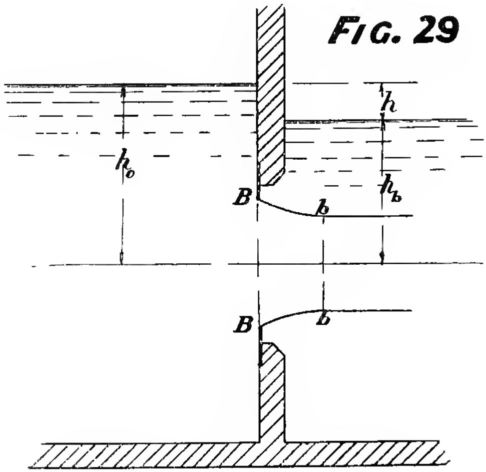
[1] At very low heads a slight error would be introduced in calculating the velocity from the head h measured to the centre of the orifice. If the theoretical velocity of each filament, due to its own depth below the water-surface, is separately calculated, the mean velocity will be a little *less* than that of the central filament. This would account for a slight *fall* in the value of the coefficient, but cannot account for the observed *rise*, which must be due to imperfect construction.

in the discharge may be taken to represent changes in the coefficient of contraction. The discharge will always be increased whenever the convergence of the stream-lines is in any way interfered with, and would be considerably augmented by any rounding or bevelling of the inner edge of the orifice.

It is also affected in some degree by any change in the temperature of the water, but further experiments are needed before this effect can be brought into calculation.

Art. 49. Flow through a Submerged Orifice.—When the water is discharged from a higher to a lower reservoir through a submerged orifice in a thin plate, as illustrated in Fig. 29, the conditions will be somewhat changed. If h_0 and h_b denote the heights

FIG. 29



at which the water stands, in the two reservoirs, above the centre of the orifice, it is obvious that the expended energy will be represented by the fall $h = h_0 - h_b$, and if this energy were wholly devoted to the work of acceleration, the theoretical velocity of the contracted jet, at the section bb, should be $v = \sqrt{2g(h_0 - h_b)}$.

The converging stream-lines will certainly approach the opening from all sides as before, and experiment shows that the contraction of the issuing jet must be very nearly the same as it was when discharging into free air. Hence the frictional resistances which were encountered in approaching and passing the sharp edge will remain unaltered, but before the sectional plane bb is reached,

the jet must now traverse the short distance *Bb* through a water lining instead of through air, so that here some work may be expended in the passage, and the coefficient of velocity at section *bb* will be somewhat smaller than in the case of free discharge.[1]

It remains only to notice that the coefficient of contraction, so far as it is affected at low heads by the cutting off of converging stream-lines, will now depend upon the height of the water-surface, in the upper reservoir, above the top of the orifice; *i.e.* it will depend upon h_0, and not upon the effective head h.

To obtain a practical comparison between the free and the submerged discharge, we cannot do better than refer to the experiments which were carried out for that purpose by Mr. Hamilton Smith with circular, square, and rectangular orifices of various dimensions.

The results given in Table 21 are reproduced from Smith's "Hydraulics," and refer to a series of gaugings made with a circular orifice, 0·10 feet in diameter, in a thin brass plate, the same plate being used in both sets of experiments. When gauging the discharge of the submerged orifice, the height of water in the lower tank above the centre of the orifice varied from 0·57 to 0·73.

The columns headed C and C_s give the coefficients of discharge for the free and the submerged jet respectively, the first being the ratio $Q \div a_0\sqrt{2gh_0}$, while the second gives the quantity $Q \div a_0\sqrt{2gh}$; and in both cases the figure includes a slight calculated correction for a minute leakage. In experiments Nos. 265 and 266, with the free discharge, the coefficient rises sharply from 0·608 to 0·620 as the water-level comes near to the top of the orifice; but in the case of the submerged jet there still remains a height h_0 of nearly 1 foot, even at the lowest head h, and the coefficient is not much increased.

[1] After passing the plane *bb*, the *whole* of the acquired kinetic energy may very likely be dissipated in the form of eddies, but this does not affect the velocity at *bb*.

TABLE 21.—DISCHARGE FROM CIRCULAR ORIFICE, 0·10 DIAMETER.

Discharge in Free Air.				Submerged Discharge.			
No.	h_0	Q	C	No.	$h = h_0 - h_b$	Q	C_s
256	4·640	0·0814	0·6014	267	3·970	0·0750	0·5992
257	4·140	0·0768	0·6017	268	3·570	0·0711	0·5987
258	3·940	0·0750	0·6018	269	2·990	0·0650	0·5989
259	3·640	0·0721	0·6020	270	2·580	0·0605	0·5997
260	3·140	0·0671	0·6024	271	2·000	0·0533	0·6006
261	2·640	0·0616	0·6027	272	1·510	0·0462	0·6006
262	1·940	0·0529	0·6036	273	0·985	0·0375	0·6025
263	1·400	0·0451	0·6055	274	0·648	0·0304	0·6027
264	1·020	0·0384	0·6080	275	0·250	0·0189	0·6048
265	0·502	0·0273	0·6148				
266	0·315	0·0216	0·6200				

From these and other experiments it is quite certain that in the submerged jet the discharge is very little less than in the free jet, while the contraction must be very nearly the same in both. The observed differences are such as would naturally result from the changed conditions which were discussed at the beginning of this article. If we compare the coefficients C and C_s at similar effective heads h, the difference between them is a quantity ranging from $\frac{1}{4}$ per cent. to nearly 2 per cent.;[1] but these aberrations disappear if we consider the contraction as depending upon the height h_0. The results observed throughout the series are then consistent with the following inferences—that for any given height h_0 the coefficient of contraction is the same in both cases; and that in both cases the coefficient of velocity is nearly constant at all heads, being about 0·97 in the free jet, and about 0·965 in the submerged jet, this difference being due apparently to the frictional work on the length Bb of submerged jet.

This would give for the frictional loss of head in the free jet, $z = 0{\cdot}062\frac{v^2}{2g}$; and in the submerged jet $z_s = 0{\cdot}074\frac{v^2}{2g}$.

In itself this figure is perhaps of little importance, but it may possess some interest in connection with the cases next to be considered.

[1] Weisbach takes the difference at the average value of $1\frac{1}{2}$ per cent.

Lastly, it may be well to notice that the opening of an ordinary sluice-gate can hardly be described as an orifice in a thin plate, and the contraction may often be suppressed by some feature in the surfaces adjacent to the opening. The coefficient of discharge will never be *less* than 0·59 or 0·60, but it may be considerably greater if the edges are splayed, or if the thickness of the gate bears a large proportion to the width of opening.

Under heads varying from $14\frac{1}{2}$ to 6 feet, the discharge from a canal sluice $4\frac{1}{4}$ feet wide, has been found to give coefficients varying from 0·594 to 0·641, the mean value being 0·625; but when the gate was only raised a few inches the coefficient reached 0·803—a fact which can best be understood by studying the somewhat analogous conditions that present themselves in the inlet next to be considered.

Art. 50. Plain Cylindrical Inlet.—In a great many cases the inlet to a cylindrical pipe is formed, as in Fig. 30, by simply allowing the cylinder to terminate at the transverse plane *AA*, which is the plane of the reservoir wall, at right angles to the axis of the pipe. But in this simple form of inlet the changes of velocity and of head are a little more complex than in any other.

In approaching this circular aperture from the interior of the reservoir, the stream-lines are free to converge from all directions outside the plane *AA*, just as in the case of an orifice in a thin plate; and the jet, as it enters the pipe, is contracted in a similar manner. Thus, if the pipe is very short in length, and is used as a discharging outlet, the contracted jet is seen to issue at the axis of the cylinder without touching its sides, just as in the case illustrated in Fig. 24, the jet, however, having a diameter equal to about eight-tenths of the bore of the pipe.

The same contraction would certainly take place if the short cylinder were submerged, and would also take place at the entrance of a long water-main; but in the latter case it is evidently impossible that a central jet can continue to travel for an indefinite distance through a stationary sheath of water. At a short distance beyond the inlet the pipe is found to run full-bore, and this is seen to be the case when the water is discharged into free air through a cylindrical mouthpiece, whose length is not less than $2\frac{1}{2}$ or 3 times the diameter, as sketched in Fig. 30.

Experiments have often been made with an outlet of such proportions, and the observed coefficient of discharge has been compared with those obtained in other outlets, such as the orifice

in the thin plate, or the conoidal outlet illustrated in Art. 44; but the cases are not really analogous, and the figures obtained in this way do not convey a sufficiently complete statement of the facts.

The sectional area a_0 of this cylindrical outlet is, of course, the same at both ends, and the quantity $\frac{Q}{a_0}$ is obviously the mean velocity V at the plane of exit *ee*. The ascertained "coefficient of discharge" is the ratio $Q \div a_0\sqrt{2gh}$, and is found to be something like 0·825 varying from 0·815 to 0·83 in the different experiments

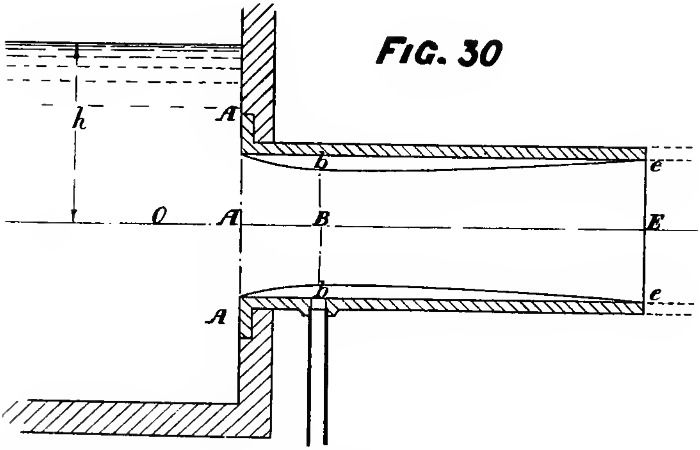


FIG. 30

of Venturi, Michelotti, Bidone, Castel, and Eytelwein, and reaching a somewhat higher figure in pipes of very small diameter.

The discharge is therefore very much greater than that which is obtained under the same measured head h from an orifice of the same diameter in a thin plate; and the fact needs some explanation.

Again, it has been pointed out that the coefficient of velocity in this case is the same thing as the coefficient of discharge, and is notably *less* than the values found in other outlets. Evidently this is true in regard to the velocity V at the plane of exit; but the facts can hardly be elucidated by any comparison between the velocity V and the measured head h.

The work of acceleration is completed in this, as in all the previous examples, when the water arrives at the plane *bb*, and it is here that we may expect to find, as in other cases, a velocity v corresponding very nearly with the energy expended.

At this contracted section the area a is about $0{\cdot}64a_0$. It follows that the velocity must here be $v = \frac{V}{0{\cdot}64} = 1{\cdot}562V$ pretty nearly; and it is now evident that this is really a *greater* velocity than could possibly be produced by the measured loss of head h. For the experiments above quoted have shown that V averages $0{\cdot}825\sqrt{2gh}$, and, multiplying by 1·562, we find that the current is running through the neck bb with a velocity $v = 1{\cdot}289\sqrt{2gh}$, which is nearly 29 per cent. *greater* than the theoretical velocity due to the head h.

The fact indicates without doubt that the effective head at the section bb is greater than the height h measured from top-water level to the centre of the jet. The velocity v cannot be produced without a fall, which is at least $\frac{v^2}{2g}$, or about $1{\cdot}66h$; and, as shown in the last article, the fall must in practice be somewhat greater than this to include frictional losses, so that it will be nearly—

$$h_b = 1{\cdot}074\frac{v^2}{2g} = 1{\cdot}7836h$$

or about $1\frac{3}{4}$ times as great as the visible head h.

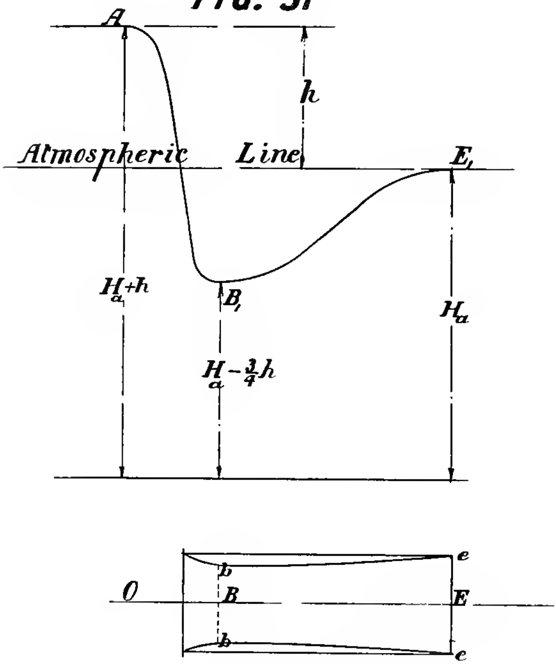
And this theoretical requirement is exactly in accordance with the observed facts, for in such experiments it is always found that a partial vacuum is maintained in the pipe at the narrowest part of the contracted vein. A vertical glass tube tt attached to the neck of the pipe at this point will draw up water from a lower level, and, when equilibrium is established, the vacuum is found to sustain a column of water whose height is proportional to h, and is generally equal to three-fourths of h very nearly.

The effect of this partial vacuum will easily be seen if we refer all the pressures or equivalent heads of water to the absolute zero. Thus, using H_a to denote the head of water which is equivalent to the barometric pressure, we shall have at the point O in Fig. 31 (above the inlet) an absolute pressure equivalent to the head $H_a + h$, while at the point B it will fall to $H_a - \frac{3}{4}h$, rising again to H_a as the jet emerges into free air at E.

The curve $A_1B_1E_1$ sketched in the figure will represent something like the piezometer readings, or the true hydraulic gradient, for the length OE along the axis of the pipe.

On passing the plane *bb*, the acceleration ceases—the sectional area is then gradually enlarged, and therefore the velocity is gradually retarded as the water moves forward from *B* to *E*. Neglecting for the moment any frictional losses, we may consider the piezometer drop from A_1 to B_1 as a measure of the work which is converted into the kinetic energy $\frac{v^2}{2g}$, and the subsequent rise from B_1 to E_1 represents the reconversion of kinetic into

FIG. 31



potential energy—a reconversion which follows here as it does in the upward swing of a pendulum.

The storing and restoring of energy are quite similar to those which take place in the Venturi water-meter; but in this cylindrical outlet the restoration cannot be so complete as it is in the water-meter, because here the water starts from a state of rest, and leaves with the velocity V possessing the kinetic energy $\frac{V^2}{2g}$, so that the restored energy is not the whole quantity $\frac{v^2}{2g}$, but only

the difference $\frac{v^2 - V^2}{2g}$, and therefore the rise B_1E_1 would be considerably less than the drop A_1B_1, even if there were no frictional losses.

But we can go on to test this question still further, for we have already calculated the vertical rise B_1E_1 as amounting in these experiments to about $0{\cdot}7836h$, and we can compare this with the kinetic energy that is given up as the water forces its way from B to E.

Expressing the quantities in terms of h, we have the initial kinetic energy at B, or $\frac{v^2}{2g} = 1{\cdot}6607h$, while the kinetic energy of the effluent current at E will be $\frac{V^2}{2g} = 0{\cdot}6806h$, so that the energy given up will be—

$$\frac{v^2 - V^2}{2g} = h(1{\cdot}6607 - 0{\cdot}6806) = 0{\cdot}9801h$$

while the actual recovery of head is $0{\cdot}7836h$. The difference, amounting to $0{\cdot}1965h$, will therefore represent the work which has been wasted during the passage from B to E, either in overcoming friction or in some other way.

This loss, which may be designated by z_2, is equivalent to $0{\cdot}118\frac{v^2}{2g}$; while the frictional loss on the length AB was found to be $z_1 = 0{\cdot}074\frac{v^2}{2g}$. The total loss is then $z_1 + z_2 = 0{\cdot}192\frac{v^2}{2g}$, which is equivalent to $0{\cdot}47\frac{V^2}{2g}$. It is thus seen that about four-fifths of the kinetic energy which had been acquired at the point B is reconverted or carried forward, the remaining one-fifth being wasted in traversing the length BE. It would be difficult to determine the actual *frictional* loss on this length, but comparing z_2 with z_1, it would appear that there is not much work wasted in any other way.

Apart from such wasted work, the conditions which govern the natural discharge from such an outlet appear to be these:—

1. The velocity v is determined by a certain effective head h_b, which is greater than the visible head h by the difference B_1E, or the barometrical depression.

2. The depression is proportional to the restored kinetic energy,

and therefore depends upon the relation between the two velocities v and V; while these velocities must always be inversely proportional to the areas a and a_e—the areas of the contracted vein and of the effluent stream respectively.

3. Of these two areas, the first is automatically determined by the natural convergence of the stream-lines at the inlet, while the second is fixed by the actual dimensions of the mouthpiece; and in this cylindrical example it happens to be exactly equal to the area a_o at the inlet orifice, so that the ratio of divergence is the same thing as the natural ratio of contraction.

If the mouthpiece were judiciously splayed out at E, so as to increase the ratio of divergence $\frac{a_e}{a}$, we might expect that a larger portion of the total kinetic energy $\frac{v^2}{2g}$ would be reconverted into head, and that the change would have the effect of increasing (within certain limits) the barometric depression, the effective head h_b, the velocity v, and the discharge Q. These effects are, in fact, observed in Venturi's experiments with diverging conical outlets, as will presently be seen.

At the same time, there is evidently a limit to the depression, which can never go below the absolute zero, so that, however great may be the head h, the effective head can never be more than $h + H_a$. Therefore, with heads greater than about 40 or 44 feet the effective head will not be so much as $1\frac{3}{4}h$, and the coefficient of discharge will be less than 0·825. Accordingly, Mr. Hamilton Smith has found that the discharge under a head of 338 feet is not much increased by the addition of a diverging outlet to the mouthpiece, for this great head would not be very notably increased by the addition of the atmospheric head H_a. On the other hand, if the whole thing were submerged, so that the discharge at E should take place under a head h_e in the lower reservoir, the depression might no doubt approximate to the magnitude $h_e + H_a$; and the same remark would apply to the inlet of a syphon working under pressure.

In the case of a long water-main, or a culvert whose length is much greater than three diameters, the discharge will, of course, not depend merely upon the head h over the inlet, but in much greater measure will depend upon the frictional resistance in the long pipe, so that the velocity V will be determined by the hydraulic gradient s, as already mentioned in Art. 8. But if

the pipe terminates at the upper reservoir in a plain cylindrical inlet, we shall have the same contraction of the entering jet, and the same changes of pressure and of velocity at the first entrance of the water, as those which take place on the short length OE of Fig. 30, although no further change of velocity can take place after passing the section ee. Hence, for any given value of V we can calculate the loss of head h_1 which takes place on the length OE; and it may conveniently be expressed by Weisbach's formula—

$$h_1 = \frac{V^2}{2g}(1 + \zeta_1) \qquad (2c)$$

in which the term $\zeta_1 \frac{V^2}{2g}$ represents the wasted head $z = z_1 + z_2$; while the value of ζ_1 can be directly found from the experiments, being equivalent to the quantity $\frac{1}{C^2} - 1$.

According to the experiments above quoted, ζ will vary from 0·41 to 0·56, and may probably be taken at 0·50 as an average, so that approximately we may write for this form of inlet—

$$h_1 = 1{\cdot}50\frac{V^2}{2g}. \qquad (2d)$$

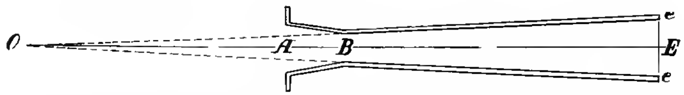
To recapitulate the successive changes, we have first, on the length OB, an effective loss of head h_B which is equivalent to the acquired kinetic energy $\frac{v^2}{2g}$ + the frictional work z_1; and secondly, on the length BE, a restoration of kinetic energy amounting to $\frac{v^2 - V^2}{2g}$ which goes partly to perform the frictional work z_2, while the remainder is reconverted into head. The transformations and retransformations cancel each other, and the final balance of results, on the whole passage OE, is that the head h has been lost, the kinetic energy $\frac{V^2}{2g}$ has been acquired, and the work $z_1 + z_2$ has been performed against frictional resistances, and we have $h = \frac{V^2}{2g} + z_1 + z_2$.

Art. 51. Conical Inlets and Outlets.—The successive changes of pressure and of velocity which were described in the last article are still more clearly illustrated and more fully developed in the flow of water through conical outlets such as those employed by Venturi.

In these experiments the water entered from a reservoir through a short converging conical inlet AB, passing immediately into a long diverging conical outlet, through which it was discharged into free air.

At every point along the line ABE in Fig. 32 the velocity will be inversely proportional to the square of the diameter, so that acceleration goes on from A to the point B, where the kinetic energy is $\frac{v^2}{2g}$. From this point the velocity is retarded, as the area of the stream is gradually enlarged in its passage to the exit at E,

FIG. 32



where the final velocity V is *very much* smaller than v, so that a *large* part of the kinetic energy is given up in the passage from B to E. Venturi found that the best results were obtained when the angle of divergence θ, or eOe, was about 5° 6′, and this angle has generally been adhered to in the Venturi water-meter, as it is now made by American engineers; but it is obvious that, without altering this angle, the area a_e at the outer end may be made to bear any desired proportion to the contracted area a, if the tube BE is made sufficiently long.

In the very instructive experiments of Mr. Francis, the long

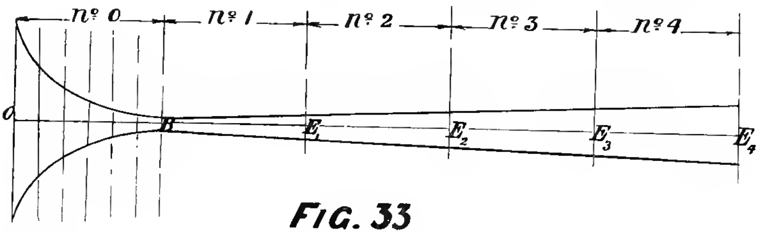


FIG. 33

diverging cone was made in four separate pieces, which could be screwed on one after the other, as illustrated in Fig. 33.

At the small end the cone was attached by a similar screwed joint to the conoidal inlet No. 0, the diameter of the contracted throat being 0·1018 ft., while at the outer end of No. 4 the cone had a diameter of 0·4085 ft.

By attaching successively the cones Nos. 1, 2, 3, and 4, the effluent stream was enlarged at the outlet to an area a_e, whose ratio to the contracted area a is given in the first column of the following table.

TABLE 22.—CONICAL OUTLETS.

	Ratio $\frac{a_e}{a}$	Ratio $\frac{V^2}{v^2}$	$\frac{v^2 - V^2}{v^2}$
No. 0	1·00	1·000	0·000
Nos. 0 and 1	2·04	0·240	0·760
Nos. 0, 1, and 2... ...	5·28	0·036	0·964
Nos. 0, 1, 2, and 3 ...	9·94	0·010	0·990
Nos. 0, 1, 2, 3, and 4 ...	16·10	0·004	0·996

The second column gives the corresponding values of the ratio $\left(\frac{V}{v}\right)^2$, which is always equivalent to $\left(\frac{a}{a_e}\right)^2$; and the figures give us the kinetic energy $\frac{V^2}{2g}$ at each joint of the pipe, in terms of the whole kinetic energy $\frac{v^2}{2g}$.

The vertical ordinates E_1e_1, E_2e_2, etc., in Fig. 34 are propor-

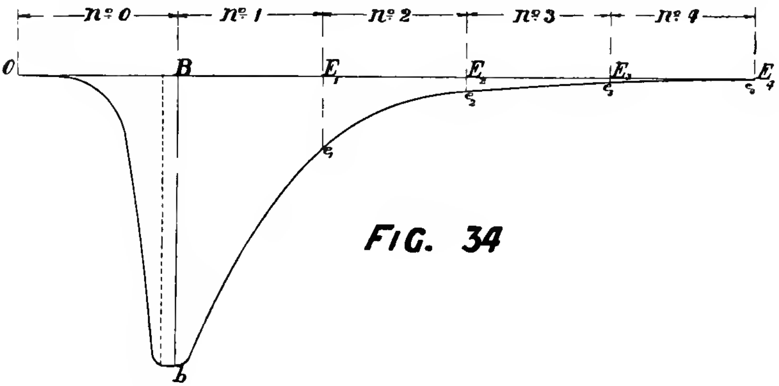


FIG. 34

tional to these figures, and may therefore represent the kinetic energy of the stream at each point in the length of the entire cone, while the curve $be_1e_2e_3e_4$ would represent the rising hydraulic gradient in such a conical outlet if there were no frictional losses.

The recovery is almost complete when the water has reached the end of the second cone, where the kinetic energy E_2e_2 is only $3\frac{1}{2}$ per cent. of the kinetic energy Bb, so that $96\frac{1}{2}$ per cent. of the whole has already been given up, and there is not much remaining to be recovered on the succeeding lengths, No. 3 and No. 4. The third column in the table gives the restored energy $\frac{v^2 - V^2}{2g}$ in terms of the whole $\frac{v^2}{2g}$.

The restoration of kinetic energy being so nearly complete, we should certainly expect to find that the discharge would be greatly augmented by the employment of this diverging outlet; and it is not at all surprising to learn that, when the conoidal inlet was prolonged by the addition of two cones, the discharge was actually doubled. Indeed, if there were no frictional losses or wasted work of any kind, we ought to have in each case a terminal velocity V equal to $\sqrt{2gh}$; and as the velocity at B must be $v = V\frac{a_c}{a}$, it follows that the coefficient of velocity at the contracted point ought to be $C_B = \frac{a_c}{a}$. In each case the coefficient would then have the value given in the first column of the table—when the whole length of cone is used, it ought to be 16; and when three lengths only are used, it ought to be nearly 10.

Now, the experiments of Mr. Francis show that the coefficient does not quite reach 2·5 in either case, and its limitation within this lower figure is really the fact that needs to be explained.

That the coefficient is greater than 1 is not at all remarkable; but we have to consider why it is only 2 or 2·5, and not 10 or 16. The matter is worth investigation if only for the sake of the Venturi meter and its calibration; and if we go on, with this end in view, to make a rough calculation of the frictional resistances, we shall find that, broadly speaking, they are enough to account for the observed facts—that they practically govern the discharge through the conical outlet—and that when they are duly allowed for, the whole of the remaining kinetic energy is reconverted into head in each case.

In passing along the tube from end to end, the frictional loss of head s per unit of length will vary from point to point, being greatest at the narrow neck where the velocity is greatest; and, therefore, the gradient which represents this frictional loss *per se*

must be a curved line, like the line Off_4 in Fig. 35; while the actual piezometer reading would be given by the curve Obe_1e_4, which is obtained by plotting the ordinates $\frac{V^2}{2g} = f_1e_1$, etc., below the curve Off_4.

The total frictional fall E_4f_4 will consist of the two portions z_1 and z_2, which are the losses of head in the inlet and in the outlet respectively, while the total fall E_4e_4 will be the observed head, or $h = z_1 + z_2 + \frac{V^2}{2g}$.

The fall z_1 was directly measured in the experiments of Mr. Francis by the gauged discharge through the conoidal inlet taken

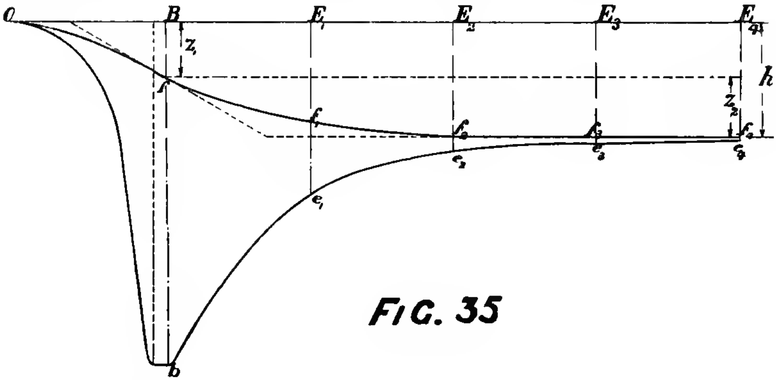


FIG. 35

by itself; but it remains to calculate the fall z_2 due to pipe-friction in the conical tube BE_4.

To compute the frictional fall in any conical pipe, Weisbach assumes, as usual, that the resistance varies with the square of the velocity w, while w varies inversely with the square of the diameter y. For any given diameter y, the gradient is, therefore, expressed by $s = \xi\frac{w^2}{2gy}$.[1]

If d is the diameter at the small end, then at any horizontal distance x measured from that end, the diameter will be $y = d + 2x \tan\frac{\theta}{2}$; so that—

[1] This is the same thing as the old formula $s = \frac{w^2}{KR}$, if we make $\frac{\xi}{2g} = \frac{4}{K}$, or $\xi = \frac{8g}{K}$.

$$w = v\frac{d^2}{y^2} = \frac{v}{\left(1 + \frac{2x}{d}\tan\frac{\theta}{2}\right)^2}$$

Taking a very short element of length δx, the slope length is given by $\dfrac{\delta x}{\cos\frac{\theta}{2}}$, and the frictional loss of head on this short element is given by—

$$\delta z = s\frac{\delta x}{\cos\frac{\theta}{2}} = \xi\frac{\delta x}{y\ \cos\frac{\theta}{2}} \cdot \frac{w^2}{2g} = \xi\frac{v^2}{2g} \cdot \frac{\delta x}{y\cos\frac{\theta}{2}\left(1 + \frac{2x}{d}\tan\frac{\theta}{2}\right)^4}$$

$$= \xi\frac{v^2}{2g}\frac{\delta x}{d\cos\frac{\theta}{2}\left(1 + \frac{2x}{d}\tan\frac{\theta}{2}\right)^5}$$

By integration this gives for the whole frictional loss of head z_2 on the length *BE*—

$$z_2 = \tfrac{1}{8}d\ \text{cosec}\frac{\theta}{2}\left[1 - \left(\frac{a}{a_e}\right)^2\right]\xi\frac{v^2}{2gd} \qquad (16)^{1}$$

That is to say, the fall z_2 is equivalent to the frictional loss of head in a cylindrical pipe whose diameter is d, and whose length is $\lambda_2 = \frac{1}{8}d\ \text{cosec}\frac{\theta}{2}\left[1 - \left(\frac{a}{a_e}\right)^2\right]$ in which the term $\left[1 - \left(\frac{a}{a_e}\right)^2\right]$ is equivalent to the fraction $\dfrac{v^2 - V^2}{v^2}$ in the third column of the table (22).

It will easily be seen that Weisbach's formula (16) may be written—

$$z_2 = \tfrac{1}{8}\xi\ \text{cosec}\frac{\theta}{2} \cdot \frac{v^2 - V^2}{2g} \qquad (16\text{A})$$

which makes the frictional loss z_2 directly proportional to the restored kinetic energy, or say—

$$z_2 = \zeta_e \cdot \frac{v^2 - V^2}{2g} \qquad (17)$$

where the coefficient ζ_e is the quantity $\frac{1}{8}\xi$ cosec $\frac{\theta}{2}$.

When the cone is formed with the particular taper chosen by Venturi, as it is in the Venturi meter, and also in the outlet of

[1] Weisbach's "Mechanik," 1ster Theil, 2te Hälfte; § 431.

Fig. 33, we should have $\theta = 5^\circ\ 6'$, and cosec $\frac{\theta}{2} = 22\cdot4$; which gives—

$$\zeta_c = 2\cdot8\xi, \text{ and } z_2 = 2\cdot8\xi\frac{v^2 - V^2}{2g} \qquad (17a)$$

And further, if we estimate K in the old formula at the common value of 10,000, or $c = 100$,[1] we shall have $\xi = 0\cdot026$, and $\zeta_c = 0\cdot072$,

$$\text{or, } z_2 = 0\cdot072\frac{v^2 - V^2}{2g} \qquad . \ (17b)$$

At the inlet *OB* we have, of course, to consider the fall z_1; and in Art. 44 we have already seen that, when the length of a well-shaped conoidal mouthpiece is about $2\frac{1}{4}d$, the loss z_1 is approximately $0\cdot05\frac{v^2}{2g}$, or say $\zeta_1 = 0\cdot05$. But the conoidal inlet of Fig. 33 is very much larger, its length being not less than $11d$, and the frictional resistance was found to be greater.

When the mouthpiece was tested for discharge, without the addition of the diverging cone, the highest recorded coefficient of velocity was 0·944, so that the frictional loss was not less than $\zeta_1 = 0\cdot122$ or $z_1 = 0\cdot122\frac{v^2}{2g}$ when the velocity was about 9·3 feet per second.

In the subsequent experiments with the diverging cone, the velocity v ranged generally between 5 and 20 feet per second, the maximum being 22; and the coefficients ζ_1 and ζ_c would probably vary a little with the velocity; but, whatever may be their exact value, we should have generally—

$$\begin{aligned} h &= z_1 + z_2 + \frac{V^2}{2g} \\ &= \zeta_1\frac{v^2}{2g} + \zeta_c\frac{v^2 - V^2}{2g} + \frac{V^2}{2g} \\ &= \frac{v^2}{2g}(\zeta_1 + \zeta_c) + \frac{V^2}{2g}(1 - \zeta_c) \end{aligned}$$

For example, in the case of the Francis experiments, if we

[1] This may be compared with the values of c found in experiments Nos. 6 to 9, and Nos. 90 to 94. The true value of c and of ξ will certainly vary with the velocity, as already shown in Chap. IV.

take the coefficients at the *average* values given above, we should have—

$$h = \frac{0 \cdot 194 v^2 + 0 \cdot 928 V^2}{2g} = \frac{v^2}{2g}\left[0 \cdot 194 + 0 \cdot 928\left(\frac{a}{a_e}\right)^2\right]$$

Hence, if the restoration of kinetic energy is really effective, and subject only to the inevitable frictional waste as above computed, the velocity v would be determined solely by the frictional loss, and would be—

$$v = \sqrt{2gh} \cdot \frac{1}{\sqrt{0 \cdot 194 + 0 \cdot 928\left(\frac{a}{a_e}\right)^2}}$$

giving a coefficient of velocity at the throat—

$$C_v = \frac{1}{\sqrt{0 \cdot 194 + 0 \cdot 928\left(\frac{a}{a_e}\right)^2}}$$

This value of the coefficient C_v, calculated for each successive length of the diverging cone, is given in the first column of the following table, while the second column gives the *average* value in each case as found by experiment.

TABLE 22A.—DISCHARGE OF CONICAL OUTLETS.

TUBES EMPLOYED.	COEFFICIENT C_v, AVERAGE VALUE.	
	Calculated. $C_v = \frac{1}{\sqrt{0 \cdot 194 + 0 \cdot 928\left(\frac{a}{a_e}\right)^2}}$	Observed.
No. 0 alone	0·944	0·934
Nos. 0 and 1	1·550	1·548
Nos. 0, 1, and 2 ...	2·096	2·090
Nos. 0, 1, 2, and 3 ...	2·220	2·299
Nos. 0, 1, 2, 3, and 4...	2·250	2·248
Infinite length[1] ...	2·270	

[1] When the cone is extended to an indefinite length the expression becomes $h = \frac{v^2}{2g}(\zeta_1 + \zeta_e)$; or $C_e = \sqrt{\frac{1}{\zeta_1 + \zeta_e}}$. If the conoidal inlet used by Weisbach were attached to the diverging cone, we might expect the limiting value to be slightly higher—probably $\frac{1}{\sqrt{0 \cdot 124}} = 2 \cdot 83$, or thereabout.

The figure given in the second column is, in each case, the average of some eight or ten experiments made under different heads. As the velocity increases the coefficient is found to show a slight rise; and this is naturally to be expected, because the resistance is not quite proportional to v^2, but to some lower power approximating $v^{1.75}$, as already seen in Art. 44.[1]

But a comparison of the average values above tabulated, and of the whole series, would appear to warrant the conclusion that, *when the cone is formed to this particular taper*, there are practically no losses, except the ordinary losses due to pipe-friction. With this inevitable drawback, the restored kinetic energy appears to be wholly converted into head.

It would not be safe, however, to extend this conclusion to all angles of taper, nor perhaps to all velocities; for the experiments themselves show that when the velocity v ran beyond 15 or 20 feet per second, the coefficient ceased to show any further rise, and began to show a slight fall—indicating, perhaps, that for any higher velocity the cone would require to have a more gentle taper, in order to prevent the contracted jet from leaving the internal surface.

As the detailed results seem to possess a special interest in connection with the use of the Venturi meter, they are given in Table 22B *in extenso*. In all cases the discharge was submerged, and the head h is, of course, the difference of level between the water-surface in the upper and in the lower reservoir. The pipes were of cast iron carefully bored, and ground to a smooth surface with emery, but without any polish or any glazed lining.

[1] If we determine the frictional loss in a conical pipe by the logarithmic formula already used for cylindrical pipes, we should have by integration—

$$h_1 = \mu\frac{v^n}{R^m} \cdot \frac{d}{2} \cdot \frac{\operatorname{cosec}\frac{\theta}{2}}{2n+m-1}\left\{1 - \left(\frac{d}{D}\right)^{2n+m-1}\right\}$$

or for smooth pipes of Class 1—

$$h_1 = \mu\frac{v^{1.75}}{R^{1.167}} \cdot \frac{d}{7.33} \cdot \operatorname{cosec}\frac{\theta}{2}\left\{1 - \left(\frac{d}{D}\right)^{\frac{11}{3}}\right\}$$

TABLE 22B.—DISCHARGE OF CONICAL OUTLETS.

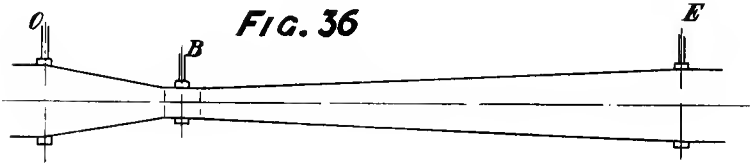
No.	Experiment.	h	C_v
276	Conoidal inlet (Francis)	0·53	0·927
277	Smallest diameter = 0·1018	0·78	0·935
278	Length, 1·10 feet, of which	0·96	0·928
279	0·10 feet is cylindrical	1·23	0·933
280		1·40	0·937
281		1·52	0·944

INLET WITH CONE NO. I.			INLET WITH CONES I AND 2.		
No.	h	C_v	No.	h	C_v
282	0·20	1·48	288	0·14	1·98
283	0·30	1·51	289	0·21	2·03
284	0·40	1·54	290	0·31	2·07
285	0·85	1·59	291	0·50	2·12
286	1·45	1·60	292	0·71	2·15
287	1·47	1·57	293	1·10	2·16
			294	1·31	2·12

INLET WITH CONES 1, 2, AND 3.			INLET WITH CONES 1, 2, 3, AND 4.		
No.	h	C_v	No.	h	C_v
295	0·13	2·08	303	0·11	2·05
296	0·27	2·25	304	0·15	2·11
297	0·44	2·26	305	0·21	2·17
298	0·63	2·30	306	0·31	2·26
299	0·92	2·38	307	0·42	2·31
300	1·18	2·43	308	0·64	2·30
301	1·36	2·43	309	0·96	2·31
302	1·39	2·26	310	1·29	2·39
			311	1·36	2·33
			312	1·41	2·25

Art. 52. The Venturi Water-Meter.—The section drawn in Fig. 36 represents the general form of the Venturi meter, and is reproduced from a modern example.[1] The taper of the outlet cone is generally 5° 6′, but the dimensions will depend upon the size of the main in which the meter is introduced. In this particular example the diameter at each end is 5 feet, while the throat has a diameter of 1 foot $10\frac{1}{2}$ inches, the total length being nearly 46 feet. The inlet, as well as the outlet, is of conical form, rounded off at each junction with the cylindrical pipe; but the inlet is a much shorter cone, the angle at the apex being about 20° 30′, so that here cosec $\frac{\theta}{2}$ is about 5·60. Between the two cones there is a short cylindrical throat, whose length l_2 is about 2 feet in this example, or say $l_2 = 1 \cdot 1d$. The conical pipes are of cast iron, the throat being smoothly bored and lined with gunmetal.

FIG. 36



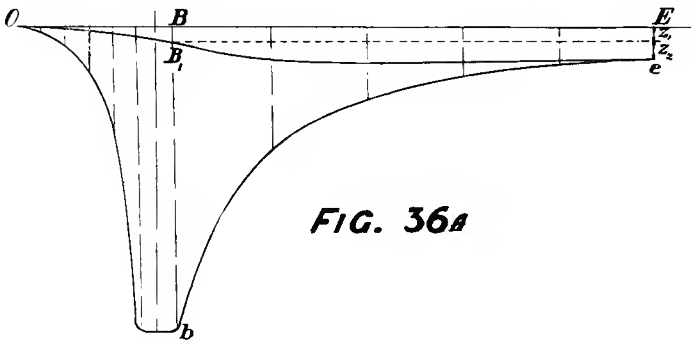


FIG. 36A

At the points O, B, and E the tube is surrounded by an annular space connected with the interior by a number of small holes; and the pressure in these annular spaces is read by three mercurial gauges. The readings in inches of mercury are easily converted

[1] This meter was constructed by the Builders' Iron Foundry, Providence, U.S.A. Vide *Engineering*, 1896, p. 207.

into feet of water, and become the vertical ordinates in the piezometer diagram, Fig. 36*a*.

From B to E the curve will be similar to that already drawn in Fig. 35 of the preceding article; and the chief difference between the two cases is that the water now enters at O with a velocity V which is *exactly the same* as the effluent velocity at E. In the experiments of Mr. Francis a large part of the energy expended in work of acceleration was restored during the passage from B to E, but here the *whole* is restored, and if there were no frictional losses there should be no fall from O to E. In other words, the observed fall Ec will represent simply the wasted work, or $Ec = h = z_1 + z_2$.

In a frictionless meter the drop H_1 from O to b would be the same as the rise H_2 from b to c, and they would represent the kinetic energy which is first taken up and then restored by the current; *i.e.* $\frac{v^2 - V^2}{2g} = H_1 = H_2$.

But in practice there will be a loss z_1 between O and B, and another loss z_2 between B and E, so that $\frac{v^2 - V^2}{2g} = H_1 - z_1 = H_2 + z_2$; and the discharge will be given quite accurately by either of the meter-readings H_1 or H_2, if the corresponding loss z_1 or z_2 can be computed or experimentally ascertained.

Thus if a is the carefully measured area of the bored neck, we have $Q = va$; while $v^2 - V^2 = 2gH_b$, where $H_b = H_1 - z_1 = H_2 + z_2$. At the same time we have—

$$v^2 = \frac{v^2 - V^2}{1 - \left(\frac{a}{a_e}\right)^2} = \frac{2gH_b}{1 - \left(\frac{a}{a_e}\right)^2}$$

so that—

$$Q = va = a\sqrt{\frac{2gH_b}{1 - \left(\frac{a}{a_e}\right)^2}}. \qquad (18)$$

To calibrate the instrument, however, it is necessary to determine either z_1 or z_2.

As a practical example on a large scale, we may refer to the recent gaugings of Messrs. Marx, Wing, and Hoskins,[1] who employed a Venturi meter differing but little in dimensions from the example illustrated in Fig. 36, and supplied by the same makers.

[1] *Proceedings Am. Soc. C.E.*, May, 1898.

The diameter at the outer end of each cone was 4 feet 6 inches, and at the cylindrical throat 2 feet $1\frac{1}{2}$ inches; so that in this example we should have—

$$\frac{a}{a_c} = \frac{3 \cdot 546}{15 \cdot 90} = 0 \cdot 223\,; \quad \left(\frac{a}{a_c}\right)^2 = 0 \cdot 223^2 = 0 \cdot 0497$$

$$1 - \left(\frac{a}{a_c}\right)^2 = 0 \cdot 9503\,; \quad \frac{1}{1 - \left(\frac{a}{a_c}\right)^2} = 1 \cdot 052$$

so that—

$$Q = va = a\sqrt{1 \cdot 052 \times 2gH_b} = 1 \cdot 026a\sqrt{2gH_b}$$

The observations recorded by Mr. Marx include the reading H_1 and the difference between the readings at O and at E, or $H_1 - H_2 = h = z_1 + z_2$; and at all velocities within the range of the experiments, they show that h is nearly proportional to H_1, or to $\frac{v^2 - V^2}{2g}$, averaging about $0 \cdot 145H_1$, as exemplified in the following table, which gives a few of the results, omitting a large number of intermediate readings:—

TABLE 22C.—LOSS OF HEAD IN VENTURI METER.

No.	Experiment.	H_1	h	$\frac{h}{H_1}$	H_2
313	Venturi Meter	0·35	0·05	0·143	0·30
314	(Marx)	0·98	0·14	0·143	0·84
315	$\frac{a}{a_c} = 0 \cdot 223$	2·00	0·29	0·145	1·71
316		2·57	0·38	0·148	2·19
317	$v^2 - V^2 = 0 \cdot 95v^2$	2·85	0·40	0·141	2·45
318		4·04	0·61	0·151	3·43
319		5·04	0·73	0·145	4·31
320		6·45	0·91	0·141	5·54

The figures show generally that the quantities h, H_1, H_2, and therefore H_b, bear the same proportion throughout or very nearly so, and that both z_1 and z_2 may be expressed as fractions of H_b.

Making $z_1 = \zeta_1 H$ and $z_2 = \zeta_c H_b$, we may write $H_1 = H_b(1 + \zeta_1)$, and $H_2 = H_b(1 - \zeta_c)$, or $H_b = \frac{H_1}{1 + \zeta_1} = \frac{H_2}{1 - \zeta_c}$.

As already seen in previous examples, the first coefficient ζ_1 will depend upon the form that may be adopted for the inlet OB

in each individual meter; while the conditions which determine ζ_c are in most cases very nearly the same as in the conical outlets of Art. 48.

It was there found that the pipe-friction on cones of varying lengths, formed to the same angle of 5° 6′ (which is the usual angle), might be broadly estimated at $0{\cdot}072\,\frac{v^2 - V^2}{2g}$ when the cone is ground to a very smooth surface, and to this must be added the frictional loss of head in the short cylindrical neck, bringing the figure up to about $0{\cdot}087\,\frac{v^2 - V^2}{2g}$, or $z^2 = 0{\cdot}087_b\mathrm{H}$.

With regard to the conical inlet, whose length is about 5 diameters, the wasted head z_1 is almost certain to be greater than in the carefully proportioned conoidal mouthpiece employed by Weisbach, and having a length of only $2\frac{1}{4}$ diameters. We may rather estimate it from the observed coefficient in Venturi's experiments with a conical outlet having the same angle of convergence, and this would give $z_1 = 0{\cdot}063\frac{v^2}{2g}$, or say $0{\cdot}066\mathrm{H}_b$. We should then have for the total wasted head *Ee*, in Fig. 36*a*, the computed value—

$$z_1 + z_2 = \mathrm{H}_b\,(0{\cdot}066 + 0{\cdot}087) = 0{\cdot}153\mathrm{H}_b$$

which, in this example, would be equivalent to $0{\cdot}143\mathrm{H}_1$.

This is almost exactly the average value of the actual loss of head $h = z_1 + z_2$, as recorded by the mercurial pressure-gauges in the experiments above quoted; and we may conclude that the known frictional resistances will account for nearly all the loss of head in the Venturi meter. When the frictional work has been allowed for, all the remainder of the kinetic energy is reconverted into head; and there can scarcely be a doubt that the same restoration would take place if the short cylindrical neck were extended to any indefinite length, or if the Venturi outlet-cone were employed as a discharging outlet at the lower end of a long water-main.

For the exact calibration of a Venturi meter, or the determination of H_b, some further experiments are to be desired; but it will certainly depend in each instrument upon the frictional resistance or roughness of the cones, and the best evidence that can be obtained on this question will be given by a comparison of the actual readings at *O* and at *E*.

The frictional losses z_1 and z_2, as above computed, would make $z_2 = 0{\cdot}57h$, and if H_b is taken at the value $\mathrm{H}_2 + 0{\cdot}60h$, the resulting discharge as calculated by formula (18) agrees pretty nearly with the discharge as computed by Mr. Marx.

But these losses are mainly, if not wholly, frictional. In a very smooth tube they will be nearly proportional to $v^{1{\cdot}75}$, and the coefficients ζ_1 and ζ_c will vary as the velocity increases; while in a rougher tube they may be more nearly constant, the friction being then nearly proportional to v_2, as they appear to be in this particular example.

To eliminate any error that might arise from variations in the frictional resistance, it would certainly be better to compute H_b by means of *both* readings H_1 and H_2, rather than by applying any assumed coefficient to the single reading H_1.

For any meter of the usual proportions, the true value of H_b cannot be far from the value $\mathrm{H}_b = \mathrm{H}_2 + 0{\cdot}6h = 0{\cdot}6\mathrm{H}_1 + 0{\cdot}4\mathrm{H}_2$; and if this value is interpolated between the actual readings H_1 and H_2, the probable error will be a very small one, whatever may be the smoothness or roughness of the individual instrument.

Art. 53. Table of Theoretical Heads and Velocities.—In all the calculations that have been referred to in this chapter, the loss of head h, or h_1, or z is some fraction or multiple of the quantity $\frac{v^2}{2g}$; and in like manner the velocity v or V is some fraction of the theoretical quantity $\sqrt{2gh}$. To make the calculations more readily, it will be convenient to make use of the following table, which gives the theoretical velocity v due to various heads up to 60 feet.

TABLE 23.—GIVING THE VELOCITY v IN FEET PER SECOND DUE TO THE FALL h IN FEET, OR $v = \sqrt{2gh}$.

h	v	h	v	h	v	h	v
0·01	0·80	0·09	2·41	0·17	3·31	0·25	4·01
0·02	1·13	0·10	2·54	0·18	3·40	0·26	4·09
0·03	1·39	0·11	2·66	0·19	3·50	0·27	4·17
0·04	1·60	0·12	2·78	0·20	3·59	0·28	4·25
0·05	1·79	0·13	2·89	0·21	3·68	0·29	4·32
0·06	1·97	0·14	3·00	0·22	3·76	0·30	4·39
0·07	2·12	0·15	3·11	0·23	3·85	0·31	4·47
0·08	2·27	0·16	3·21	0·24	3·93	0·32	4·54

TABLE 23—*continued.*

h	v	h	v	h	v	h	v
0·33	4·61	0·98	7·94	3·00	13·90	11·00	26·62
0·34	4·68	1·00	8·03	3·10	14·13	11·50	27·21
0·35	4·75	1·04	8·18	3·20	14·36	12·00	27·80
0·36	4·81	1·08	8·34	3·30	14·57	12·50	28·40
0·37	4·87	1·12	8·50	3·40	14·80	13·00	28·90
0·38	4·94	1·16	8·64	3·50	15·01	13·50	29·50
0·39	5·01	1·20	8·78	3·60	15·22	14·00	30·00
0·40	5·07	1·24	8·94	3·70	15·44	14·50	30·50
0·42	5·20	1·28	9·08	3·80	15·64	15·00	31·10
0·44	5·32	1·32	9·22	3·90	15·84	15·50	31·60
0·46	5·44	1·36	9·36	4·00	16·05	16·00	32·10
0·48	5·56	1·40	9·50	4·25	16·54	16·50	32·60
0·50	5·67	1·44	9·62	4·50	17·02	17·00	33·10
0·52	5·79	1·48	9·74	4·75	17·49	17·50	33·60
0·54	5·90	1·52	9·88	5·00	17·94	18·00	34·00
0·56	6·00	1·56	10·02	5·25	18·39	19·00	35·00
0·58	6·11	1·60	10·14	5·50	18·82	20·00	35·90
0·60	6·22	1·64	10·28	5·75	19·24	21·00	36·80
0·62	6·32	1·68	10·40	6·00	19·66	22·00	37·60
0·64	6·42	1·72	10·52	6·25	20·06	23·00	38·50
0·66	6·52	1·76	10·64	6·50	20·46	24·00	39·30
0·68	6·61	1·80	10·76	6·75	20·85	25·00	40·10
0·70	6·71	1·84	10·88	7·00	21·23	26·00	40·90
0·72	6·81	1·88	11·00	7·25	21·61	27·00	41·70
0·74	6·91	1·92	11·12	7·50	21·98	28·00	42·50
0·76	7·00	1·96	11·24	7·75	22·34	29·00	43·20
0·78	7·09	2·00	11·36	8·00	22·70	30·00	43·90
0·80	7·18	2·10	11·64	8·25	23·05	32·00	45·40
0·82	7·26	2·20	11·90	8·50	23·40	34·00	46·80
0·84	7·35	2·30	12·17	8·75	23·74	36.00	48·10
0·86	7·44	2·40	12·44	9·00	24·07	38·00	49·50
0·88	7·53	2·50	12·69	9·25	24·41	40·00	50·80
0·90	7·61	2·60	12·94	9·50	24·73	45·00	53·80
0·92	7·70	2·70	13·18	9·75	25·06	50·00	56·80
0·94	7·78	2·80	13·42	10·00	25·38	55·00	59·50
0·96	7·86	2·90	13·67	10·50	26·00	60·00	62·20

Intermediate values can easily be obtained by interpolation, and any increment of kinetic energy can be found by simple subtraction. Thus, if V_e is 40 feet per second, while V is 10 feet per second, we have from the corresponding values of h, $\frac{V_e^2 - V^2}{2g} = 25 - 1{\cdot}56 = 23{\cdot}44$ feet, for the fall equivalent to this change of kinetic energy.

CHAPTER VII.

LINES OF WATER CONVEYANCE.

Art. 54. The Total Loss of Head.—In the foregoing chapters we have examined the several items of work that are included in the conveyance of water by a line of conduit, and we now have to bring them together; for in any such line of water-conveyance we have always to consider the total loss of head.

We may regard the conduit as designed for the transmission of water from one quiet reservoir to another, and if the outlet is submerged the total fall H will be simply the vertical height between the water-surfaces in the two reservoirs; but if the stream is discharged in free air, and not against pressure, the fall will, of course, be measured from the upper water-surface down to the centre of the discharging orifice or outlet.

In either case the fall H will then be a measure of the whole energy expended in the journey, and it will be the algebraical sum of two or three quantities.

At the inlet there will be the first drop h_1 expended mainly in work of acceleration, and measured by the expression $\frac{V^2}{2g}(1 + \zeta_1)$, where V is the velocity of the current in the barrel.

The second item will be $h_2 = sl_2$, or the fall on the gradient s, which represents either the actual inclination of an open channel or the hydraulic mean gradient in a pipe or syphon; and in either case the drop h_2 measures the energy expended in frictional work on the whole length l_2 of the conduit beyond the inlet.

And now, if the water is finally discharged, at the same velocity V, through a plain cylindrical outlet, there will be no further losses or gains to be considered, and the total fall will be given by—

$$H = h_1 + h_2$$

—as in either of the examples which are illustrated further on by Figs. 37 and 38.

But if the pipe or culvert terminates in a diverging outlet, like that of the Venturi meter, so that the velocity V is gradually reduced to a much lower effluent velocity V_e, the greater part of the first drop h_1 will be recovered again at the outlet. If this recovery of head is denoted by h_3, the total fall from one reservoir to another will be given by—

$$H = h_1 + h_2 - h_3$$

and the piezometer diagram for such a case is illustrated in Fig. 41.

On the other hand, if the final discharge takes place through any kind of contracted outlet, or through the orifice of a partially closed sluice-valve, so that the effluent velocity V is very much *greater* than the velocity V in the pipe, there will be a further loss of head at the outlet, and if h_3 denotes this loss the total fall will be—

$$H = h_1 + h_2 + h_3 \quad . \quad . \quad (19)$$

as illustrated in the piezometer diagram of Fig. 42.

This last formula may therefore be applied to all cases if we understand that the third item h_3, representing the change of head at the outlet, may be either positive, or zero, or negative, according to the particular form of the discharging outlet.

When the form of the culvert is determined, each of the three items may be calculated from the velocity V, and their sum will give us the total fall H which must be present if that velocity of current is to be maintained.

Or again if H represents the known difference of level, or total fall between two given reservoirs, it will always be made up of the same three items, automatically adjusted in their due proportions, and the problem will be to find the velocity V.

Art. 55. Culvert or Pipe with Plain Cylindrical Ends.—The piezometer diagram for this very common case is sketched in Fig. 37. At the inlet, we have already seen in Art. 50 that it takes the form $A_1B_1E_1$, the last-named point being two or three diameters from the entrance. From E_1 to the outlet at E_2, the velocity remains constant, and the straight line E_1E_2 represents the hydraulic gradient whose fall on the length l_2 is the vertical height h_2. The gradient terminates at the water-surface in the lower reservoir, and the total fall H is the sum of the heights h_1 and h_2. From the experiments of Art. 50, it appears that we may take the loss of head up to the point E_1 as having the value—

$$h_1 = \frac{V^2}{2g}(1 + \zeta_1) = 1{\cdot}50\ \frac{V^2}{2g}\ \text{approximately.}$$

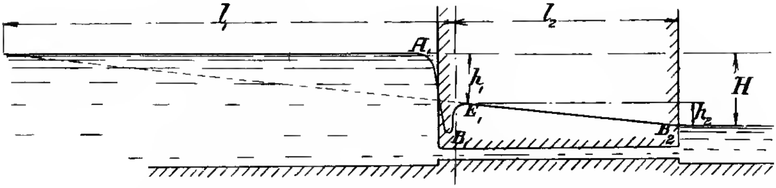
Beyond this point we have the gradient E_1E_2. The length l_2 of this falling gradient might be taken as the total length of the culvert, or more exactly perhaps at a slightly smaller value two diameters less than the outside length from face to face. Its inclination, as given by the old formula, would be $s = \frac{V^2}{KR} = \frac{V^2}{2g} \cdot \frac{\xi}{d}$; and therefore the fall h_2 on the length l_2 would be $h_2 = \frac{V^2}{2g} \cdot \xi\ \frac{l_2}{d}$.

Hence the total fall, as given by Weisbach's very convenient formula, would be—

$$H = h_1 + h_2 = \frac{V^2}{2g}(1 + \zeta_1 + \xi\ \frac{l_2}{d}) \qquad . \qquad (20)$$

But in this expression the coefficient ξ will not be a constant

FIG. 37



quantity. It will vary with the velocity of the current, the diameter of the culvert, and the roughness of its surface. Its most common value may be roughly 0·026; but it will sometimes be as small as 0·013, and at other times as large as 0·052.

With greater precision we may calculate that $h_2 = l_2\ \mu\ \frac{V^n}{R^m}$, where μ, n, m are taken at the values already found in Chap. IV.; and the total fall will then be given by—

$$H = h_1 + h_2 = \frac{V^2}{2g}(1 + \zeta_1) + l_2\mu\frac{V^n}{R^m} \qquad (21)$$

EXAMPLE.—A circular culvert 8 feet in diameter and 500 feet long is built of ordinary brickwork, and with plain cylindrical ends. The maximum flood discharge is estimated at 800 cubic feet per second, and the water is led away in a waste channel of ample dimensions. Assuming that the tail-water will not rise above the centre of the outlet, it is required to find the height to which the flood would rise at the upper end of the culvert.

The sectional area of the 8-foot barrel is very nearly 50 square feet; and the velocity must therefore be $V = \frac{800}{50} = 16$ feet per second. From this value of V we can calculate both h_1 and h_2; but if we proceed to determine the latter by means of the old formula, using an ordinary value for the coefficient ξ, we shall be likely to over-estimate the fall. For the experimental evidence which was discussed in a previous chapter shows that in a culvert of such large diameter, worked at so high a velocity, the coefficient will be far below its average value.

Referring, however, to Table 3 of Art. 28, we may take for ordinary brickwork the values $n = 1{\cdot}80$, $m = 1{\cdot}20$, and $\log \mu = \bar{5}{\cdot}975$, and by means of formula (21) we should have the following results:—

$$
\begin{aligned}
\text{Taking } l_2 \text{ at } 500 - 16 &= 484 \text{ feet.} \\
\text{Log } l_2 &= 2{\cdot}684845 \\
\text{Log } \mu &= \bar{5}{\cdot}975 \\
1{\cdot}8 \log V &= 2{\cdot}167416 \\
&\ 0{\cdot}827261 \\
-1{\cdot}2 \log R &= -0{\cdot}361236 \\
\text{Whence } \log h_2 &= 0{\cdot}466025, \text{ or } h_2 = 2{\cdot}93 \text{ feet}
\end{aligned}
$$

But at the upper end we still have to consider the drop h_1, which is far from being a negligeable quantity. To generate the velocity V, the net dynamic head is theoretically $\frac{V_2}{2g} = 4$ feet, nearly; and for the cylindrical inlet we may take h_1 at $1{\cdot}50$ times this quantity, or say 6 feet. Hence the total fall from head to tail is—

$$
\begin{aligned}
H &= h_1 + h_2 \\
&= 6{\cdot}00 + 2{\cdot}93 = 8{\cdot}93 \text{ feet}
\end{aligned}
$$

The example illustrates a calculation that may often be required in connection with the flood discharge of a large culvert, and it shows the importance of the item h_1 in many cases of this kind.

Art. 56. Culvert with Conoidal Inlet.—In constructing the large culverts which often form an essential part of storage reservoirs, and which furnish perhaps the only means of discharging the flood-waters during the execution of an embankment, it has been the practice of engineers to design the entrance in the form of a conoidal or trumpet-shaped inlet, which is generally executed in massive ashlar masonry.

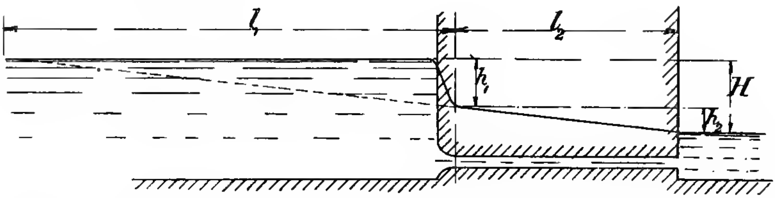
The reason for adopting this form of construction is sufficiently illustrated by the calculation worked out in the preceding article.

In that example at least two-thirds of the total flood-rise H was due to the head h_1 at the inlet; and in Art. 44 it was seen that the head $h_1 = \frac{V^2}{2g}(1 + \zeta_1)$ would not be greater than $1 \cdot 05 \frac{V^2}{2g}$ if the inlet were truly formed to the curves of the contracted vein. In that case the piezometer diagram would resemble the curve drawn in Fig. 38, and the total head H for the given discharge would work out at $4 \cdot 2 + 2 \cdot 93 = 7 \cdot 13$ feet, or nearly 2 feet less than with the plain cylindrical inlet.

In practice the entrance can hardly be formed as a complete reproduction of the conoidal inlet; for if the reservoir is to be drained by the culvert the invert must be laid at a level which is not higher than the floor of the reservoir.

But the curved section of a contracted vein can be carried

FIG. 38



round the arch and side walls of the entrance, so that on all sides, including the floor, the stream-lines are led into the culvert without giving room for the formation of a contracted jet that shall be anywhere smaller than the section of the culvert. The elevation and longitudinal section sketched in Figs. 39 and 40 may serve to illustrate approximately the form of the masonry surfaces, but the design must, of course, be adapted in each case to the cross-section of the culvert. If the curves are carefully set out, we should probably be justified in assuming that the water flows past the section *bb*, running full bore with a velocity which is maintained throughout the passage, and that the loss of head h_1 up to this point will scarcely exceed $1 \cdot 05 \frac{V^2}{2g}$. But the true value of the loss, z_1, will certainly depend upon the configuration of the entrance. If the best proportions are not attained in the design, ζ_1 may be greater than $0 \cdot 05$, and with some rounded inlets Weisbach found

$\zeta_1 = 0{\cdot}085$. On the other hand, if the surfaces follow the curves of the converging stream-lines, the loss, z_1, appears to be mainly due to skin-friction. In this case it will be nearly proportional to V^n, and at high velocities ζ_1 may be less than 0·05, as it was in the very small mouthpiece of Table 20, p. 131. At the same time it will probably be inversely proportional to R^m, and therefore in the case of an 8-foot culvert it is likely to be considerably less than in the small mouthpiece referred to in that table; while it will be influenced again by the roughness or the smoothness of the masonry surface.

In the absence of further experimental evidence the engineer

FIG. 39

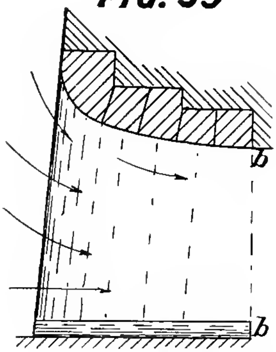
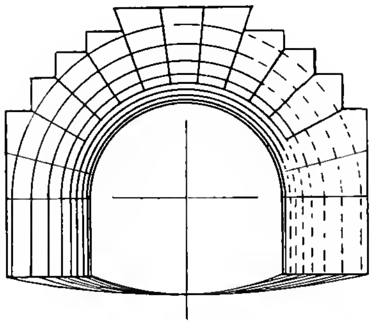


FIG. 40



can only base his calculation upon an assumed value for ζ_1 which is well on the safe side.

Art. 57. Long Lines of Water-main.—The relative importance of the two quantities h_1 and h_2 will depend very much upon the length of the conduit.

The calculation worked out in Art. 51 showed that the drop h_1 at the inlet formed two-thirds of the total fall in the case of a masonry culvert whose length was about 60 diameters; but in a long line of gravitation main the loss of head, h_2, due to frictional work forms almost the whole of the total fall H, and the drop h_1 becomes such an insignificant percentage of the whole that engineers will generally leave it out of account altogether. Indeed, the working velocity in a gravitation main is seldom as high as 8 feet per second, so that the dynamic head $\frac{V^2}{2g}$ is not often as great as 1 foot, and frequently amounts only to a few inches,

while the total fall on a long line of inverted syphon—*i.e.* the difference of level between the two reservoirs—may often be measured in hundreds of feet.

Suppose, for example, that a distance of 10 miles intervenes between a couple of break-pressure reservoirs, A and B, which are to be connected by a 42-inch main; and it is required, perhaps, to find the total fall required in order that a certain supply of 25,400,000 gallons per day may be conveyed through the main.

Referring to Example No. 2 in Art. 33, it was there found that a water-main laid with 42-inch pipes of the smoothest class would deliver the required discharge of 47·2 cubic feet per second if the hydraulic gradient s were 1 in 700, which is equivalent to a fall of 7·54 feet per mile.

Hence if we leave the item h_1 out of account, we may estimate the total fall at the value $h_2 = 10 \times 7{\cdot}54 = 75{\cdot}40$ feet.

However, it was seen in the course of the calculation that, with the given discharge, the velocity must be $V = 4{\cdot}9$ feet per second; and referring to Table 23 in Art. 53, we find at once that the dynamic head $\frac{V^2}{2g}$ amounts to 0·38 feet. Taking $1\frac{1}{2}$ times this quantity for a plain cylindrical inlet, we have $h_1 = 1{\cdot}5 \times 0{\cdot}38 = 0{\cdot}57$ feet. It might be somewhat more correct, therefore, to reckon the total fall at the value $75{\cdot}40 + 0{\cdot}57$, or say 76 feet; but the correction is so small that it is not worth the making.

There is indeed nothing to be gained by striving to correct small inaccuracies in a calculation of this kind, where errors of much greater magnitude are almost sure to be present. And this is especially true in all calculations which relate to the free discharge of long water-mains.

Art. 58. Cylindrical Pipe or Culvert with Conical Outlet.—We have seen that in conduits of moderate length, worked at any high velocity, the chief part of the fall H will be the drop h_1 at the inlet; and in Articles 51 and 52 it was shown that this drop h_1 might be almost wholly recovered at the outlet by a reconversion of kinetic energy into head.

It is probable that this recovery would best be effected by a uniform retardation of the current in a discharging outlet of *conoidal* form. The diverging mouthpiece might then be shorter and the frictional losses would be less than in the conical outlet of the Venturi meter. But the experiments of Mr. Francis, and the later measurements of Mr. Marx, have shown conclusively how much of the

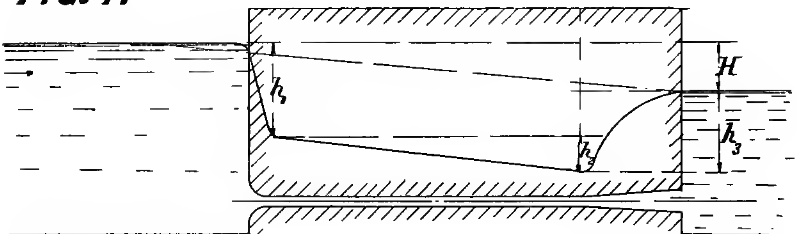
lost head *may* be recovered even by using a conical form of outlet.

If the upper end of the pipe is fitted with a conoidal inlet, the piezometer diagram must take the form sketched in Fig. 41, and may be constructed by inserting the straight gradient between the descending and ascending curves of Fig. 35, or by adding the ascending curve as sketched at the end of the gradient in Fig. 41.

The total fall from one reservoir to the other will be given by $H = h_1 + h_2 - h_3$, or $H = h_2 + (h_1 - h_3)$; and the difference $h_1 - h_3$ will be equivalent to the fall h in Fig. 35, and will be equal to $z_1 + z_2 + \frac{V_c^2}{2g}$, where z_1 and z_2 are the losses of head due to wasted frictional work on the inlet and outlet respectively.

In the preceding articles these items of wasted head have been expressed by $z_1 = \zeta_1 \frac{V^2}{2g}$, and $z_2 = \zeta_c \frac{V^2 - V_c^2}{2g}$. For an inlet $2\frac{1}{4}$ diameters in length, it was shown by experiment that ζ_1 is

FIG. 41



about 0·05; while the value of ζ_c, as estimated by Weisbach's friction formula, was about 0·072, and was found to agree very fairly with the experiments of Francis and with the larger gaugings of Mr. Marx.

For the sake of illustrating the flow of water through a conduit of such form, we may use these approximate values in working out an example:—

EXAMPLE.—Suppose the pipe shown in Fig. 41 to have a total length of 90 feet from end to end, with an internal diameter of 24 inches throughout the cylindrical portion, but with a conoidal inlet $2\frac{1}{2}$ diameters in length, and a conical outlet splaying out to a diameter of 4 feet at the mouth, the length of the cone being 21 feet. This would give the same angle of divergence as in the

Venturi meter, and there would remain 64 feet of cylindrical pipe between inlet and outlet cone.

What will now be the total fall H, if the pipe has to deliver 3000 cubic feet of water per minute?

Solution.—Taking a pipe of the smoothest class, we need not here stop to apply the logarithmic formula, as the tables will give us all that we want.

The discharge being equivalent to 50 cubic feet per second, while the sectional area of the pipe is 3·14 square feet, the velocity must be $V = 50 \div 3{\cdot}14 = 16$ feet per second nearly; and at the outer end of the cone, where the diameter is twice as great, the effluent velocity will be reduced to one-fourth, or $V_e = 4$ feet per second.

To find the head theoretically due to these velocities we may turn to Table 23, which shows that when $V = 16$ the kinetic energy for each pound of water is nearly 4 foot-pounds, or $h = 4{\cdot}0$ feet nearly; while the effluent water, leaving at the speed of $V_e = 4$ feet per second, possesses still the kinetic energy of 0·25 foot-pounds, having restored the kinetic energy $\frac{V^2 - V_e^2}{2g}$ $= 4{\cdot}0 - 0{\cdot}25 = 3{\cdot}75$ foot-pounds for each pound of water.

At the inlet, however, the drop h_1 which is given by $\frac{V^2}{2g} + z_1$ will be something greater than 4·0 feet, owing to the frictional loss $z_1 = 0{\cdot}05 \times 4{\cdot}0 = 0{\cdot}20$, and will amount to $4{\cdot}0 \times (1 + 0{\cdot}05)$ $= 4{\cdot}20$ feet.

To find the vertical fall h_2 on the straight gradient whose length l_2 is 64 feet, we may refer to Table 12, which shows that the 24-inch pipe will deliver 3000 cubic feet per minute (or a little more) when the hydraulic gradient s is 1 in 45, so that the frictional loss of head h_2 will be very nearly $64 \div 45 = 1{\cdot}42$ feet.

Up to this point, therefore, the loss of head will be $4{\cdot}20 + 1{\cdot}42$ $= 5{\cdot}62$ feet; but now we have to subtract the recovered head h_3. The restored kinetic energy is equivalent to 3·75 feet of head, but in passing along the cone a part of this is expended in frictional losses amounting to $z_2 = 0{\cdot}072 \times 3{\cdot}75 = 0{\cdot}27$ feet, so that the actual recovery will be only $3{\cdot}75 \times (1 - 0{\cdot}072) = 3{\cdot}75 - 0{\cdot}27$ $= 3{\cdot}48$ feet.

The total fall H from one reservoir to the other may, therefore, be estimated at $5{\cdot}62 - 3{\cdot}48$, or $H = h_1 + h_2 - h_3 = 2{\cdot}14$ feet.

If the 24-inch pipe had been carried out for the whole length of 90 feet with plain cylindrical ends, the total fall H for the same discharge could not have been less than $h_1 + h_2 = (4{\cdot}0 \times 1{\cdot}50) + (90 \div 45) = 6 + 2 = 8$ feet, or nearly four times as great.

Some further experiments are undoubtedly needed to determine the actual losses in diverging outlets, but it is probable that a still better result might be obtained by the use of a carefully formed conoidal outlet.

Art. 59. Water-main discharging through a Sluice-valve.—The discharge of a long pipe may sometimes be regulated and governed by a screw-down sluice-valve applied at the discharging end, as sketched in Fig. 42.

By partially closing the valve the stream is made to pass through a narrow orifice whose sectional area a_o is adjustable, and will sometimes be very much smaller than the sectional area a of the pipe itself. The effluent velocity V_e will then, of course, be *greater* than the velocity V in the bore of the pipe, and instead of any recovery there will be at the outlet a further *loss* of head h_3 due to the increased kinetic energy of the current.

The discharge of water through the valve will take place under conditions somewhat similar to those which were considered in Art. 48, where the jet issued through an orifice in a thin plate. The jet will suffer some further contraction outside the orifice, so that its actual section a_e may be reduced to about $0{\cdot}64\ a_o$; but whatever the coefficient of contraction may prove to be, the effluent velocity *of the jet* will evidently be $V_e = \frac{Q}{a_e} = V\frac{a}{a_e}$; and the loss of head at this outlet may be taken at $h_3 = \frac{V_e^2 - V^2}{2g}(1 + \zeta_3)$, where ζ_3 is about $0{\cdot}06$, as shown experimentally in Art. 48.

The piezometer diagram must, therefore, take the form sketched by way of illustration in Fig. 42.

The total fall will now be the sum of the three items,

$$\text{or, } H = \quad h_1 \quad + h_2 + \quad h_3$$

$$= \frac{V^2}{2g}(1 + \zeta_1) + sl_2 + \frac{V_e^2 - V^2}{2g}(1 + \zeta_3) \qquad (22)$$

and if r denotes the ratio $\frac{V_e}{V} = \frac{a}{a_e}$, this becomes—

$$H = sl_2 + r^2\frac{V^2}{2g}\left\{1 + \zeta_3 + \frac{\zeta_1 - \zeta_3}{r^2}\right\} \qquad (22a)$$

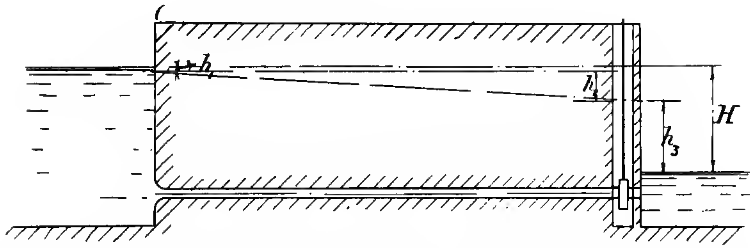
When the pipe has a conoidal inlet ζ_1 is nearly the same as ζ_3, and practically speaking—

$$H = sl_2 + r^2\frac{V^2}{2g}(1 + \zeta_3)\,. \qquad (23)$$

EXAMPLE.—Suppose that the 24-inch pipe has a length of 1000 feet, and is formed with a conoidal inlet. But suppose also that the discharge is partly choked by a screw-down valve at the lower end, so that the effluent jet is reduced to an area a_e which is one-fourth the area of the pipe. What, then, will be the total loss of head if the discharge is 630 cubic feet per minute?

To find the gradient on the length of 1000 feet we may use Table 12 if the pipes are of the smoothest kind, and it gives us 1 in 700 for a discharge of 630 cubic feet per minute in a 24-inch pipe. Hence the fall h_2 will be $1000 \div 700 = 1·43$ feet. To find the other losses, we have $Q = 630 \div 60 = 10·5$ cubic feet per second, while the area of the pipe is $a = 3·14$ square feet, so that the velocity in the pipe will be $10·5 \div 3·14 = 3·34$, while that of

FIG. 42



the effluent jet will be four times as great, or $V = 3·34$ and $V_e = 13·36$.

Turning then to Table 23, we find $\frac{V^2}{2g} = 0·172$, while $\frac{V_e^2}{2g} = 2·77$, and $\frac{V_e^2 - V^2}{2g} = 2·60$ nearly. Hence the first drop will be only $h_1 = 1·05 \times 0·172 = 0·19$; but there will be a greater loss of head at the partially closed valve, amounting to $1·07 \times 2·60 = 2·78$ feet; and the total fall will be $H = 0·17 + 1·43 + 2·78 = 4·38$ feet.

Art. 60. Calculations of Discharge or Velocity.—In many cases the data for these calculations are simply the dimensions of the conduit, and the total fall H from one reservoir to the other. When the length l_2 of a water-main is very great, the fall h_2 is

so large in comparison with the initial drop h_1 that the latter may often be left out of account, and the calculation would then be made by the methods described in Chap. V. But we have already seen that this omission would lead to very great errors in calculating the discharge of a culvert of moderate length; for here the drop h_1 may sometimes form the greater part of the total fall H. We must, therefore, find the true gradient s by subtracting the drop h_1 as illustrated in Figs. 37 and 38, so that the remainder h_2 may represent the actual fall due to frictional resistance.

In each of those two cases, however, it is evident that we may arrive at the same result by adding, at the upper end of the culvert, a certain extra length, l_1, such that $l_1 s$ shall be equal to h_1, and then the true gradient will be—

$$s = \frac{\mathrm{H}}{\mathrm{L}} \text{ or } \frac{\mathrm{H}}{l_1 + l_2}$$

If we might only assume that s is proportional to V^2, a direct solution could easily be obtained by Weisbach's formula, $\mathrm{H} = \frac{\mathrm{V}^2}{2g}\left(1 + \zeta_1 + \xi\frac{l_2}{d}\right)$, which gives—

$$\mathrm{V} = \sqrt{\frac{2g\mathrm{H}}{1 + \zeta_1 + \xi\frac{l_2}{d}}} \quad . \quad . \quad . \quad (24)$$

and this might be further simplified for practical use by introducing the value of L, which could easily be found if the coefficient ξ were really constant.

Thus, if we assume ξ to have the constant value 0·025, it will follow that a fall equal to $\frac{\mathrm{V}^2}{2g}$ upon the gradient must be subtended by an extra length equal to just 40 times the diameter of the culvert; so that in every case we should have $l_1 = 40d\,(1 + \zeta_1)$; and adding this extra length, or making $\mathrm{L} = l_1 + l_2$, we come to the old formula—

$$\mathrm{V} = \sqrt{\frac{2g}{\xi}} \cdot \sqrt{d} \ \sqrt{\frac{\mathrm{H}}{\mathrm{L}}} = \sqrt{\mathrm{K}\ \frac{\mathrm{D}}{4} \cdot \frac{\mathrm{H}}{\mathrm{L}}}$$

But unfortunately ξ is not constant, and the true discharge cannot be so simply calculated. Of the two elements which go to make up the total fall H, the first h_1 is certainly proportional to V^2, while it is equally certain that h_2 is proportional to another power of V. Hence it is impossible to express the velocity V as

any simple function of the total fall H; but for practical purposes the calculation can be effected by trial and error as follows:—

Taking provisionally $l_1 = 40d(1 + \zeta_1)$ as a rough estimate for the extra length, find $L = l_1 + l_2$, and for the assumed gradient $\frac{H}{L}$ calculate the corresponding value of V by the method of Art. 33. Calling this the first value of V, find the drop $h_1 = \frac{V^2}{2g}(1 + \zeta_1)$, and the fall $h_2 = H - h_1$. Then calculate again the velocity from the gradient $\frac{h_2}{l_2}$; and if this second value of V coincides nearly with the first, it may be accepted. Otherwise repeat the calculation of h_1 to obtain a closer value of h_2 and of V.

EXAMPLE.—A straight pipe of the smoothest class, 24 inches in diameter and 1000 feet long, forms a line of conveyance from one reservoir to another, the total fall from one to the other being 5 feet. What will be the discharge if the pipe is formed with plain cylindrical ends, the inlet and outlet being both submerged?

For a submerged cylindrical outlet we have $h_3 = 0$, and, therefore, $h_2 = H - h_1$. Also we have seen that an inlet of plain cylindrical form gives $h_1 = \frac{V^2}{2g} \times 1{\cdot}50$ nearly.

If we ventured to ignore the drop h_1, we should have $s = \frac{5}{1000} = 1$ in 200, and for this gradient the logarithmic formula would give us a velocity of very nearly 7 feet per second. Then, the area of the pipe being $2^2 \times \frac{\pi}{4} = 3{\cdot}1416$ square feet, the discharge would come to nearly 22 cubic feet per second, or more nearly 1304 cubic feet per minute, as given directly in Table 12A. But this rough calculation would evidently over-estimate the effective gradient s.

On the other hand, if we take Weisbach's formula (24), assuming $\xi = 0{\cdot}025$, the extra length l_1 would be $40d \times 1{\cdot}50 = 120$ feet, so that $L = 1000 + 120 = 1120$ feet, and the effective gradient would be $\frac{H}{L} = \frac{5}{1120} = 1$ in 224. The calculated velocity according to this formula would then be—

$$V = \sqrt{\frac{2 \times 32{\cdot}2}{0{\cdot}025} \; 2 \cdot \frac{5}{1120}} = \sqrt{23} = 4{\cdot}8 \text{ feet per second}$$

but this assumed value of the coefficient would over-estimate the resistance of such a pipe.

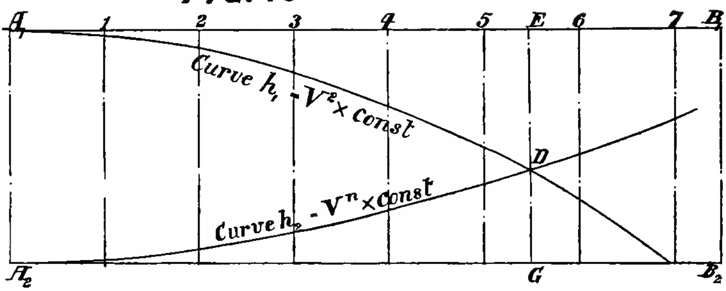
To come more closely to the truth, we may begin with the gradient of 1 in 224 as an approximation, and by the method of Art. 33 we have—

$$\text{Log V} = \tfrac{2}{3} \log \text{R} + \tfrac{4}{7} (\log s - \log \mu) = 0{\cdot}8117$$

which gives 6·48 as the first approximation for V With this velocity we have $h_1 = 1{\cdot}50 \times \dfrac{6{\cdot}48^2}{64{\cdot}4} = 0{\cdot}979$, or $h_2 = 5{\cdot}0 - 0{\cdot}98 = 4{\cdot}02$ nearly; and the calculated gradient is now more like 1 in 250. For this lower value of s the velocity may be calculated again by the same formula, and we get a second approximation, V = 6·11 instead of 6·48. A third trial gives the gradient at 4·1 in 1000, and V = 6·22 instead of 6·11, and the true quantity lies between these two values.

By repeated trial the figure may be determined within any

FIG. 43



required limits of error; but it may be obtained more directly by a graphic method, in which the two items h_1 and h_2 are represented by separate ordinates. Thus in Fig. 43 set off to any convenient scale of feet the height A_1A_2, to represent the total fall H, and draw the two parallel lines A_1B_1 and A_2B_2. On these lines set off a convenient scale of velocities, and for the successive values V = 1, 2, 3, 4, etc., calculate h_1 and h_2, setting off h_1 from the line A_1B_1 downwards, and h_2 from the line A_2B_2 upwards. The first will be a common parabola in which every ordinate is $h_1 = \text{V}^2 \times \dfrac{1 + \zeta_1}{2g}$; while the ordinates of the other curve will be $h_2 = \text{V}^n \times \dfrac{\mu l_2}{\text{R}^m}$. The two curves will intersect each other at a

certain point D, and through this point draw the vertical EDG. Then ED will represent the drop h_1, while DG gives the remaining fall h_2, and the length $AE = A_2G$ represents the true velocity V. The figure is drawn to scale for the solution of the example just above treated.

The ordinates are found without much labour if the values of V^n are taken out of the following table, and multiplied by the constants :—

TABLE 24.—CALCULATED VALUES OF V^n.

V	$V^{1\cdot75}$	$V^{1\cdot77}$	$V^{1\cdot80}$	$V^{1\cdot90}$	$V^{1\cdot96}$	V^2
1	1·00	1·00	1·00	1·00	1·00	1
2	3·36	3·41	3·48	3·73	3·89	4
3	6·84	6·99	7·23	8·12	8·61	9
4	11·32	11·63	12·13	13·93	15·14	16
5	16·72	17·26	18·12	21·30	23·44	25
6	23·00	23·80	25·20	30·10	33·50	36
7	30·10	31·30	33·20	40·30	45·30	49
8	38·10	39·60	42·20	52·00	57·60	64
9	46·80	48·90	52·20	65·00	74·20	81
10	56·20	58·90	63·10	79·50	91·20	100
11	66·40	69·70	74·90	95·20	110·00	121
12	77·30	79·90	87·60	112·30	130·40	144

The above values of n are those comprised in Table 3.

Art. 61. Calculation of Choked Discharge.—When the outlet is commanded by a screw-down sluice-valve, the discharge may, of course, be arbitrarily reduced by partially closing the valve, and the conditions will be those illustrated in Fig. 42 of Art. 59.

The total fall H will be made up of three items, h_1, h_2, and h_3, but may again be divided into two parts, of which the one $(h_1 + h_3)$ is proportional to V^2, while the other h_2 is proportional to V^n. The case may therefore be treated by the graphic method which has just been described, and it is worth while to notice the manner in which the discharge of such a water-main is really governed by the adjustable aperture of the valve. In the case of a very short pipe, where h_2 becomes an inconsiderable item, the velocity V_e of the effluent jet under the nearly constant head h_3 will be nearly constant, and the discharge will be approximately proportional to the opening of the valve; but this is very far from

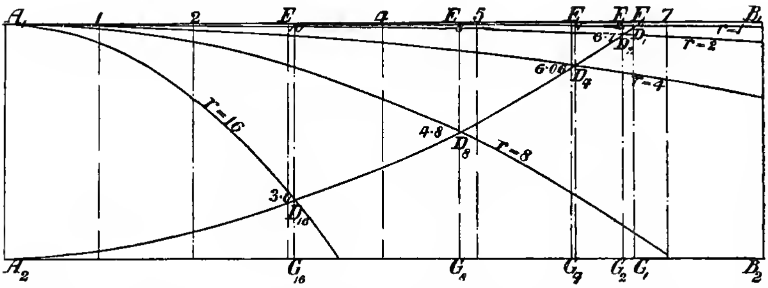
being true when the valve commands the lower end of a long water-main, and to illustrate the conditions which then arise we may take the following example:—

EXAMPLE.—Taking a similar 24-inch pipe, suppose its length to be l_2 = 10,000 feet, while the total fall is H = 50 feet. At the upper end let the pipe be fitted with a conoidal inlet, and at the lower end with a screw-down sluice-valve, by which the area of the effluent stream a_e is first made equal to a, the full area of the pipe, and is then successively reduced to $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, and $\frac{1}{16}$ of a. For each setting of the valve, what will be the discharge, or what will be the velocity V in the pipe itself?

(The discharge Q will be simply Va, and it will be enough to find V.)

To construct the graphic diagram which is drawn to scale in Fig. 44 we have to find the constants for the two curves, whose

FIG. 44



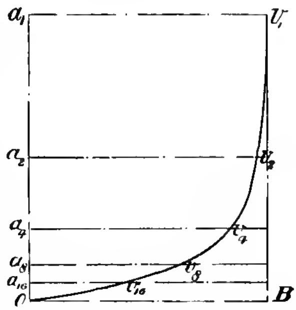
ordinates give the quantities $(h_1 + h_3)$ and h_2 respectively. Turning then to formula (23) of Art. 59, it will be seen that $h_1 + h_3 = r^2 \cdot \frac{V^2}{2g}(1 + \zeta_3)$, in which $(1 + \zeta_3)$ is about 1·07; and the constant will therefore be $r^2 \times \frac{1 \cdot 07}{64 \cdot 4} = \frac{1}{60} r^2$, where r is the ratio $\frac{a}{a_e}$. Taking r, therefore, at the successive values 2, 4, 8 and 16, we can readily find the constant which has to be multiplied by V^2 to give the ordinates for each of the parabolic curves, starting from A_1 in the diagram.

Next we have to deal with the fall h_2 due to frictional resistance on the length l_2, and it will be proportional to V^n, being in every case $h_2 = sl_2 = l_2 \cdot \frac{\mu}{R^m} = \frac{10{,}000\mu}{R^m}$, where $R = \frac{d}{4} = 0 \cdot 5$ feet, $m = \frac{7}{6}$, and $\log \mu = \bar{5} \cdot 878$. Working it out by logarithms we have very easily $h_2 = 1 \cdot 7V^n$ very nearly, and taking the values

of $V^{1\cdot75}$ from Table 24, the ordinates of the curve starting from A_2 are readily found, and are drawn to scale in the figure.

When the valve is fully open, or $r = 1$, the loss h_3 is, of course, nothing, and the drop h_1 is only about 1 foot, as shown by the height E_1D_1, so that h_2 is about 49 feet, and the velocity V appears to be 6·8 feet per second as nearly as it can be scaled. It will be noticed that this maximum velocity is not sensibly reduced by closing the valve down to one-half the area, nor very much reduced when the valve leaves an opening of only one-fourth, the velocity being 6·7 in the first case, and about 6·06 in the second. And when the valve is almost shut down, leaving only an aperture one-sixteenth of the full area, the velocity is still 3 feet per second, and the discharge not much less than one-half of the maximum.

FIG. 45



The relation between the variable opening of the valve and the consequent discharge may be conveniently represented by the co-ordinates of a diagram such as Fig. 45, in which the heights Oa represent the variable areas of the effluent jet, while the lengths av give the corresponding velocities of the current in the pipe, so that they also represent on another scale the discharge of the water-main. The shape of the curve Ov_8v_1 will depend chiefly on the length of the pipe. If it were indefinitely short the curve would be a straight inclined line; but the longer the pipe the more nearly does the curve approach to the lines OB and Bv_1.

EXAMPLE 2.—The question just treated might have been put in another form as follows: With the given water-main and the given fall H, it is desired to limit the discharge to a certain fixed quantity—say $9\frac{1}{2}$ cubic feet per second, so that the velocity V is to be 3 feet per second and no more. By how much, then, must the aperture of the valve at the outlet be reduced?

The answer to this question can evidently be given by direct calculation. For a velocity of 3 feet per second we find from Table 24 that $V^{1\cdot75} = 6\cdot84$, and we have already seen that in this example $h_2 = 1\cdot7V^{1\cdot75} = 1\cdot7 \times 6\cdot84 = 11\cdot63$ feet. Hence it follows that $h_1 + h_3 = 50\cdot0 - 11\cdot63 = 38\cdot37$; and it has also been calculated

that $h_1 + h_3 = \frac{V^2 r^2}{60}$, so that we have $r^2 = \frac{60 \times 38{\cdot}37}{3 \times 3} = 255{\cdot}8$; or $r = 16$ very nearly. The valve must therefore be screwed down until the effluent jet is reduced to one-sixteenth of the full area of the pipe.

Art. 62. Working Pressure.—The fluid pressure at any point in a gravitation main is most conveniently expressed and measured in feet of head, whatever may be the varying conditions which affect that pressure. Thus we may understand h to denote the vertical height of a column of water equivalent to the actual working pressure at any given point, and when we have calculated h we shall have the working pressure in pounds per square foot, or $p = \mathrm{h}\gamma$. To determine h, however, we have to remember the various losses of head which have been discussed in Art. 8, and in the various cases already referred to in this chapter.

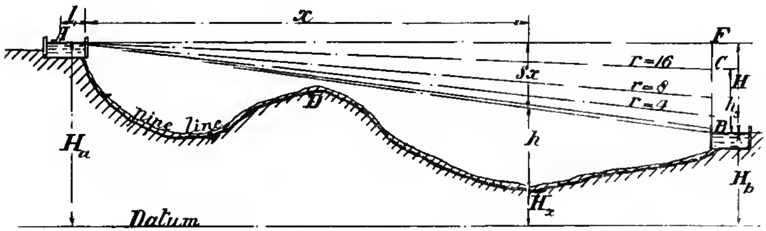
In all these cases we have supposed the pipe to be in hydraulic communication with a high-level reservoir from which it is fed; and if the water were at rest in the pipe, the effective head h at any given point would be simply the "hydrostatic head," or the vertical height measured from the level of top-water in the reservoir to the given point in the pipe. To express it with an almost unnecessary definiteness, we may write H_a for the "reduced level" of top-water, or its height above any given datum, and H_x for the "reduced level" of any given point in the pipe; the "hydrostatic head" is then $\mathrm{H}_a - \mathrm{H}_x$, and is precisely the effective head h for the given point.

But when the water is allowed to flow through the pipe, the working pressure is changed, and the corresponding head h is everywhere less than the hydrostatic head $\mathrm{H}_a - \mathrm{H}_x$. If a pressure-gauge whose readings are in feet of water were applied to the pipe just below its junction with the high-level reservoir, the gauge would register a head $\mathrm{h} = \mathrm{H}_a - \mathrm{H}_x - h_1$, in which H_x is the reduced level, and h_1 the initial drop $\frac{V^2}{2g}(1 + \zeta_1)$. Or, more generally, if we select *any* point in the pipe whose distance from the inlet (measured along the pipe) is x, the pressure-gauge applied at that point will register a head $\mathrm{h} = \mathrm{H}_a - \mathrm{H}_x - (h_1 + sx)$; where s is the hydraulic gradient equal to $\mu\frac{V^n}{R^m}$.

All this is so clearly to be seen upon the longitudinal section that it was hardly worth while to write out the algebraical

expression; the several quantities are shown by way of illustration in Fig. 46, but the summation is effected in the simplest possible way, and without any calculation, by scaling off the vertical height intercepted between the pipe and the hydraulic gradient *AB*. The "hydraulic *mean* gradient" is drawn as a straight line from one reservoir to the other, and for our present purpose this involves no considerable error, although the slope length of the pipe should, no doubt, be taken in preference to its horizontal projection x when the longitudinal section is used for other purposes. It may also be observed that the gradient should properly start from a point *A* moved back from the inlet by a short distance l_1, varying from 30 to 60 diameters; but this small correction becomes insignificant when the section extends over many miles of country. For practical purposes the measured height h will

FIG. 46



indicate the pressure which the pipe will have to bear under the ordinary working conditions.

But it is important to remember that the losses h_1 and h_2 depend upon the velocity V. The fall of the gradient will be uniform if V is uniform throughout the whole distance, and the straight gradient will terminate at the level *B* if the submerged outlet is discharging *freely* into the reservoir at the height H_b above datum; but anything that reduces the velocity of flow will reduce the slope s of the hydraulic gradient, and when V is nothing, the inclination s will also be nothing. The section will be greatly altered, therefore, whenever the flow is checked at the outlet by partially closing the valve. How the hydraulic gradient is affected by such a choking of the flow is clearly to be seen in Fig. 42 and in the calculations of Art. 61. As the valve is gradually screwed down the hydraulic gradient in Fig. 46 will gradually rise from the position *AB* to some such line as *AC*, the height *CB* being the

loss of head h_3 due to acceleration of the jet through the reduced aperture; and when the flow is stopped altogether, the line must rise to the horizontal position AF.

It is evident that the working pressure at some parts of the pipe might thus be considerably increased, and would be indicated by the head h, measured up to the higher gradient. For any given case, the question may be easily dealt with by the graphic method described in Art. 61. The diagram drawn in Fig. 44 refers to a 24-inch main, with a fall of 50 feet on a total length of 10,000 feet; and if the main whose section is shown in Fig. 46 has the same dimensions and the same fall, the several positions of the hydraulic gradient can at once be obtained by setting off the heights h_3 on the vertical line BF, those heights being taken directly from the diagram, where they are indicated by the lines E_1D_1, E_2D_2, etc., in Fig. 44.

We may use for the working head the general expression, $\mathrm{h} = \mathrm{h}_0 - h_1 - sx$, taking h_0 to denote the hydrostatic head $\mathrm{H}_a - \mathrm{H}_x$; but we must remember that the gradient s in this formula depends upon the working velocity V upon the length of pipe x. If V is itself known or determinable, then $s = \mu\frac{\mathrm{V}^n}{\mathrm{R}^m}$; and in the particular case that we have just been considering—

$$s = \frac{h_2}{l_2} = \frac{\mathrm{H} - h_1 - h_3}{l_2}$$

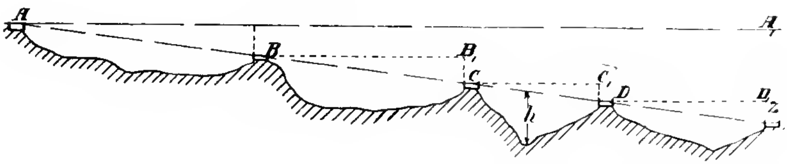
Art. 63. Break-Pressure Reservoirs..—In works of water-supply the town mains are generally fed from a service reservoir, placed at a sufficient elevation and in close proximity to the town, while the service reservoir is itself supplied from a source which may be many miles distant—or many scores of miles.

If such a long line of water-conveyance is to consist of a pipe of uniform diameter throughout, the hydraulic mean gradient must be one straight line from end to end, like the line AZ in Fig. 47; and, under the ordinary working conditions, the pipe will be subject to a pressure measured by the head h, or the depth of the pipe below the line AZ. The route may be so chosen as to bring the pipe over some intermediate hill-tops, such as B, C, D, etc., where the ground rises to the line AZ; and at each of those points the working pressure, under ordinary conditions of steady flow, would evidently be zero, or rather would be exactly equal to the atmospheric pressure. At these points the pipe might be

connected with an open tank, and the water would stand in each tank at the level of the gradient *AZ*.

In engineering practice a long water main will often be thus divided into sections by the construction of intermediate reservoirs at selected sites where the ground rises to the hydraulic gradient, and the reservoirs may serve several useful purposes. So long as the flow is maintained at the normal velocity due to the total fall upon the total length, it is obvious that the introduction of these reservoirs will not in any way affect the working pressure at any point in the main. The syphon which connects the two reservoirs *C* and *D* for example, will be subject to the same working head h as though the pipe had been continuous from *A* to *Z*. But if the discharge were at any time checked by partially closing an outlet valve at *D*, or by any obstruction in the pipe, the increased height h could now only extend to the horizontal line CC_1, even if the valve were wholly closed. And for any partial closing of the

FIG. 47



valve we could estimate the head h by measuring the length x from the reservoir *C*. The section in Fig. 47 includes only four syphons, but it is enough to show how the subdivision of a very long main into numerous sections will prevent the occurrence of working pressures which might otherwise be excessive. The pipes of each syphon may be designed with only sufficient strength to resist safely the hydrostatic head measured to topwater of the reservoir immediately above it.

The surface line sketched in Fig. 46 illustrates a case which sometimes arises. At a point *D* the pipe-line passes over a summit which does not quite reach the gradient. There are, of course, two alternative methods by which a break-pressure reservoir may be here introduced—the one is by the building of a water-tower on the top of the hill, if there is no ground in the vicinity at an elevation that would reach the gradient; the other is by depressing the gradient-line. In the latter case we should simply have two syphons, *AD* and *DB*, in which the hydraulic gradient is not the

same. The actual gradients s_1 and s_2 for the two syphons would be determined by the features of the section, and it would be necessary to calculate separately the required diameter of each syphon, in order that they may give one and the same discharge. This problem has already been dealt with in Art. 34.

It remains to be noticed that, whether the chain of reservoirs is laid upon one continuous gradient, or the fall broken up into two or more gradients, the discharge of each syphon will be governed by conditions which do not always appear in the calculations. The local occurrence of sedimentary deposit, or the presence of air in the pipe, may cause an unexpected diminution in the discharge of one syphon as compared with that of its neighbour. But it is obvious that the resulting changes of level in the intervening reservoirs will in every case tend to bring about an automatic readjustment of the gradients, and an equalization of the unequal discharges, provided that a sufficient range of level is allowed for in the break-pressure reservoirs. Wherever the obstruction may lie, the greatest discharge through the whole line of conveyance will be obtained by allowing this automatic adjustment to operate as far as may be practicable.

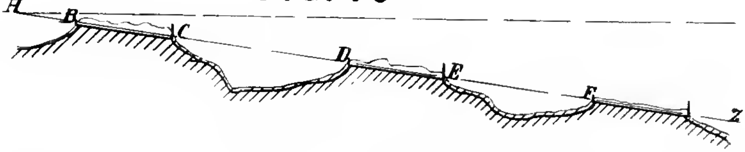
Art. **64. Syphon and Surface-Conduit.**—Wherever the ground follows pretty nearly the level of the desired hydraulic gradient, it is evident that the flow might be carried on in a pipe subjected to *no* hydrostatic pressure, or in a channel conveying the supply as a stream with an open surface. If such a route could be found, it would entirely avoid the occurrence of high working pressures, such as those which take effect in deep syphons, and it might dispense with the employment of iron or steel pipes. Constructed in this way, the line would be less costly, even though the route were a little longer. It is seldom possible to follow this method from one end to the other of a long line of aqueduct, unless the valleys are crossed by long lines of arches as in the aqueducts of old Rome; but it is often possible to select a route along which the ground keeps nearly to the desired gradient for certain distances or sections, leaving between them valleys which can be crossed by syphons, or hills which can be tunnelled. The section of the line will then take such a form as that sketched in Fig. 48, where the surface-conduits, *BC*, *DE*, etc., are laid at a level and inclination coinciding with the general gradient *AZ*, while the syphons, *CD*, *EF*, etc., convey the water from one of these conduits to the next across the intervening valleys.

This method has been adopted in great works of water-supply, like those constructed for the towns of Birmingham and Manchester, the total length of aqueduct being about 73 miles in the one case, and 96 miles in the other—the Manchester line consisting of 30 syphons, and as many intermediate stretches of surface-conduit.

The same plan is indeed often followed in works of very much smaller dimensions, and the surface-conduit may in such cases consist of a line of socket-jointed stoneware pipes. If the diameter of these surface-pipes is the same as that of the syphons, the hydraulic gradient being one straight line *AZ*, they will run full bore, and the conditions which govern the velocity and the discharge present nothing that calls for special remark.

But the tunnels and surface-conduits in a great system of water-supply will generally resemble the culverts or the open channels of Chap. III., lined with masonry or concrete, and conveying a stream which fills them to a certain height; while they

FIG. 48



may be arched over at the top, and sometimes executed in "cut and cover." The sectional area of the stream is therefore a variable quantity depending on the discharge, and with the sectional area the hydraulic radius R will also vary. In many cases the diameter of the culvert (and its radius R when running full) will be very much greater than that of the pipes which constitute the intervening syphons, the culvert being designed to carry a supply which shall be adequate for all future requirements of the town, while each syphon may consist of one or two lines of pipe sufficient for immediate needs.

All these circumstances will, of course, have some considerable effect upon the flow of the current, and the conditions which govern the flow are not the same for the syphon as they are for the culvert. So long as the syphon consists only of one or two lines of pipe, while the culvert is designed to carry the discharge of a much greater number, we may probably consider the discharge of the whole aqueduct as being governed by the calculable discharge of

the syphon upon the given gradient. The depth of the stream in the culvert will then adapt itself to that discharge, and can be found if required by the tentative method described in Art. 40. The formulæ referred to in previous articles will also suffice to calculate the maximum discharge of the culvert, and as it will generally be adapted for carrying off something *more* than the maximum discharge of the completed syphons, the supply will again be limited by the latter.

But it will hardly be quite correct, at any period, to treat the current as though it were flowing at one unchanging velocity along its whole course. The journey from end to end is not one journey at constant speed, but is broken up into stages with a considerable change of velocity at the end of each stage, and possibly a total loss of kinetic energy. For each stage we ought really to consider the initial loss h_1, and the final loss or possible recovery h_3. All these losses of head will have to be subtracted from the total fall H, and they will be repeated as many times as there are syphons in the whole length of aqueduct. What those losses would amount to must depend in each case upon the form given to the junction, and the change of velocity which takes place there. In the case of a large break-pressure reservoir we may consider that the water comes to rest, and starts from that condition when it enters the inlet of the next syphon, so that the journey through each syphon and the attendant losses of head may be treated as already described in Arts. 54 to 61.

Art. 65. Knees, Bends, and Junctions.—Wherever the direction of flow is suddenly or rapidly changed, we may expect to find a loss of head proportional to V^2, and this appears to be generally confirmed by the experiments of Weisbach, who expresses the loss of head in terms of the kinetic energy, or—

$$h_4 = \zeta_4 \frac{V^2}{2g} \tag{25}$$

in which ζ_4 is another coefficient determined by experiment.

When the water is drawn off from a **T** junction, as in Fig. 49, we have almost a repetition of the plain cylindrical inlet, through which the stream enters in converging lines with a certain contraction of area, so that if the pipe were a very short one, the issuing jet would not fill the bore; and the same thing was observed by Weisbach in the case of a square knee, although the pipe, in Figs. 49 and 50, is found to run full at a distance of two or

three diameters from the angle, just as in the plain cylindrical inlet referred to in Art. 50.

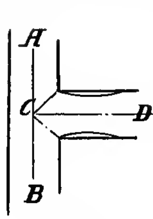
Using V to denote the velocity in the *full* pipe, the value of ζ_4 in Weisbach's experiments is from 1 to 1·5, so that we may safely estimate the loss of head at $h_4 = 1{\cdot}5\frac{V^2}{2g}$, as in the cylindrical inlet.

But this would only apply to a *sharp* angle of 90°, and would certainly be less if the junction or knee were rounded off at the corners; and the loss of head will of course be less if the meeting angle θ is less than 90°, as in Fig. 50A.

In this case Weisbach proposes to determine the coefficient ζ_4 by the formula—

$$\zeta_4 = 0{\cdot}9457\left(\sin\frac{\theta}{2}\right)^2 + 2{\cdot}047\left(\sin\frac{\theta}{2}\right)^4$$

For circular bends which run round an arc of 90°, as in Fig. 50B, the coefficient is found to depend upon the ratio between the



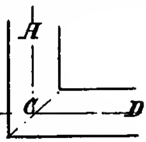


FIG. 50

FIG. 49

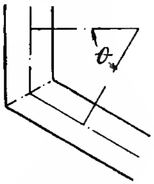


FIG. 50A

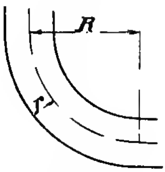


FIG. 50B

radius of curvature R and the radius of the pipe r. For different values of the ratio $\frac{r}{R}$, Weisbach gives the coefficient ζ_4 as follows:—

TABLE 25.—FOR CIRCULAR BENDS.

Ratio $\frac{r}{R}$	0·100	0·200	0·300	0·400	0·500	0·600
Coefficient ζ_4 ...	0·131	0·138	0·158	0·206	0·294	0·440

These figures may, perhaps, suffice for the rough estimation of the loss of head due to the bends upon any line of water-main; but they are deduced from old observations, and some further experiments are much to be desired.

Art. 66. The Filling of Lock-Chambers.—The *time* that it would take to lock a vessel up or down through a given height of lift h_0 is sometimes an important question in the working of ship canals, and in the design of sluices or penstocks.

In filling or in emptying the lock, the flow will take place through submerged sluices, and under a head which is constantly decreasing. The quantity of water that must be abstracted from the upper reach every time that the lock is used, will depend upon the displacement of the vessels, and upon the order in which they may arrive for passage upwards or downwards; but it may easily be seen that the quantity which has to be drawn off through the sluices is independent of the size of the vessel, and must always be equal to the vertical lift h_0 multiplied by the superficial area A of the lock-chamber—supposing, of course, that the chamber is bounded by vertical walls and vertical gates. In a chamber of this usual form it is evident that every foot of lift must require a certain number of cubic feet of water, equal to the number of square feet in the area A.

When the sluice-gate is first opened for filling the lock, the water outside will be standing at the height h_0 above the surface of the water inside; and whatever may be the depth of the submerged opening, the effective head will be h_0, and the contracted jet will flow with the velocity $V_o = 0{\cdot}97\sqrt{2gh_0}$ very nearly, as shown in Art. 49.

If a is the sectional area of the contracted jet, the discharge in cubic feet per second will now be $V_o a$, and the water will be rising inside the lock at a rate proportional to this discharge. In the first second of time it will rise, in fact, to the height $V_0 \dfrac{a}{A}$ But the filling will proceed more slowly as the water rises. When it has risen to any variable height x, the effective head will have the value $h = h_0 - x$, and the velocity of the entering jet will be $V = 0{\cdot}97\sqrt{2gh}$. At all times, however, the discharge will be Va, and the rate at which the water rises in the lock will always be proportional to V and, therefore, proportional to $\sqrt{2gh}$. We may regard it, indeed, as a velocity v in the vertical direction, proportional always to the velocity of the incoming jet; and the progressive rise of the water-surface through the whole height h_0, will be governed by conditions similar to those which determine the progressive rise of a water-particle in the vertical jet of Fig. 3, the rate of progress being proportional in both cases to the varying

quantity $\sqrt{9gh}$. Hence the time required for completely filling the lock is theoretically equal to the time required for the ascent of the jet through the same height, multiplied by the ratio $\frac{A}{a}$. In both cases the mean rate of progress is exactly one-half of the initial rate; and the total time in seconds is exactly twice the time that would have been required if the initial velocity had been maintained constant.

The contraction of the jet will, of course, depend upon the form of the inlet, and if C_a is the coefficient of contraction, we shall have for the area of jet $a = C_a a_0$, where a_0 is the area of the orifice, and the maximum discharge will be nearly $Q_0 = 0{\cdot}97 C_a a_0 \sqrt{2gh_0}$. Or if we take a suitable coefficient of discharge C, we may write $Q_0 = Ca_0\sqrt{2gh_0}$.

The total cubic contents of the lock will be Ah_0, and the time required for filling or emptying it will be $t = \frac{2Ah_0}{Q_0} = \frac{2A\sqrt{h_0}}{Ca_0\sqrt{2ag}}$.

EXAMPLE.—Let us suppose a pair of sluice-gates with square orifice 4 feet × 4 feet, delivering a submerged flow into a lock-chamber whose length from gate to gate is 300 feet, with an average internal width of 40 feet. What time will it take to fill the chamber if the lift is 16 feet?

Solution.—If the sluices are formed in the lock-gates without any splayed inlet, we may suitably take $C = 0{\cdot}6$; and with an effiective head of 16 feet, the maximum rate of discharge would then be—

$$Q = 2 \times 4^2 \times 0{\cdot}6\sqrt{2g} \times \sqrt{16} = 616{\cdot}32 \text{ cubic feet per second,}$$

while the mean rate would be one-half of this, or 308·16 cubic feet per second.

The capacity of the chamber to be filled is $300 \times 40 \times 16 = 192{,}000$ cubic feet; and the time required to fill it is, therefore, $192{,}000 \div 308{\cdot}16 = 623$ seconds, or a little more than ten minutes.

In such a case as this the problem is hardly affected at all by any question of frictional resistance; but if the discharge had been carried through sluice-passages in the masonry of the dock walls, it would have been necessary to bring the frictional losses into account.

CHAPTER VIII.

NATURAL WATERCOURSES.

Art. 67. The Flow of Rivers.—Upon most of the Continental rivers, and many in India and America, observations have been repeatedly taken to determine the measurable quantities V, R, and s; and if the flow of rivers depended upon any constant relations between these quantities, the recorded measurements should suffice to discover those relations.

But it has already been mentioned that when these results are compared together, they seem to exhibit nothing so clearly as their entire want of consistency one with another. Thus, taking the old formula $V = c\sqrt{Rs}$, as something to work with, the coefficient c has been found for each experiment, and its value varies from about 12 to 254, although it generally lies somewhere between 50 and 180.

It is easy to see, however, that there are many reasons which would go far to explain such divergences:—the sectional area of the stream is not always constant along the measured reach, and therefore the fall s may not be a true measure of the frictional work, because it may often include an unknowu element due to acceleration, positive or negative—the velocity V, which should, of course, be the *mean* velocity at all parts of the section, can scarcely be ascertained with any great accuracy—and therefore the hydraulic depth R may be the only dimension free from ambiguity. But if the measurements were ever so carefully rectified, the flow would still be very greatly affected by the character of the river-bed, and here we may certainly expect to find differences greater than any that we have found between the smoothest aud the roughest of artificial channels—differences which we can never measure or express by any mathematical quantity. It may at once be said, therefore, that we can never venture to use the gaugings of one river to calculate the flow in any other river—

still less to calculate the discharge in any regular artificial channel.

The most notable results that have been obtained by river-gaugings are those presented in the very careful measurements of Messrs. Humphreys and Abbot upon the deep waters of the Mississippi. Broadly speaking they have shown that a current flowing with a mean velocity of 4 or 5 feet per second is maintained upon a long stretch of river, where the fall is so little that it can scarcely be measured at all. The two extremest examples are those quoted in Table 26, under Experiments Nos. 339 and 340; and they bring out the coefficient c in the old formula at the remarkable values of 235 and 254—figures which are far higher than anything that has been recorded in the smoothest of pipes or artificial channels.

In reference to these two examples, it has been pointed out by other writers that no reliance can be placed upon the exact value of the slope s when the total fall, as measured, amounts only to about half an inch on the measured length of 2 miles. The smallest ripple at one end of the stretch would be enough to bring the coefficient up to infinity by obliterating the gradient altogether.

Without discarding these results, as some writers have done, we may at least beg leave to allow for a *possible* error of levelling amounting say to 0·01 feet per English mile or 1 in 500,000; and when this is done the two extreme examples fall very well into line with the other Mississippi gaugings.

Through all these experiments, however, we have a similar velocity of current maintained with very little fall, and a coefficient c which appears to grow larger as the gradient becomes flatter.

Giving expression to this observed fact, Messrs. Humphreys and Abbot proposed to modify the old formula by making $V \propto s^{\frac{1}{4}}$ instead of $\sqrt{s}$, or by writing $s \propto V^4$; but it was at once seen that this could not be applied to pipes and artificial channels, for here we have the accumulated evidence of many experiments which all go to show that s is proportional to some power of V *less* than the square, and never greater, unless it be in the roughest of dry rubble channels as in Classes 13, 14 or 15, where the exponent n may perhaps be 2·1.

Thus the great river seemed almost to stand out as a solitary example contradicting everything that had been learned from

smaller streams. The remarkably high values of the coefficient c were believed to be connected with the unusual flatness of slope, and, to calculate the widely varying values of that coefficient in the old formula, Messrs. Ganguillet and Kutter proposed to employ a *secondary* formula which is somewhat complex in shape. The coefficient, in the case of large channels, is made to increase with a *decrease* of slope, while in small channels it increases with an *increase* of slope, and is determined by calculating—

$$c = \frac{a + \frac{\beta}{\delta} + \frac{\eta}{s}}{1 + \left(a + \frac{\eta}{s}\right)\frac{\delta}{\sqrt{R}}} \qquad (26)$$

The constants a, β, η are found by reference to recorded experiments, while δ remains as a quantity which must depend in each individual case upon the roughness of the channel.

In metrical measure the formula for V becomes—

$$V = \left\{ \frac{23 + \frac{1}{\delta} + \frac{0{\cdot}00155}{s}}{1 + \left(23 + \frac{0{\cdot}00155}{s}\right)\frac{\delta}{\sqrt{R}}} \right\} \sqrt{Rs} \qquad (27)$$

Reverting, however, to the evidence of the Mississippi gaugings, it may be noticed that the appearance of incongruity is mainly due to the employment of the old formula, and to the assumption that V must be proportional to $\sqrt{R}$, or $s \propto \frac{1}{R}$.

The examination of Bazin's results in Chap. V. afforded sufficient grounds for believing that s is proportional to a power of $\frac{1}{R}$ which is always greater than unity. The power m was never less than $\frac{7}{6}$, and was found to increase with the roughness of the channel, rising to $\frac{3}{2}$ in the case of Group V., where the roughness was greatest.

For such materials as coarse gravel or rubble the formula would become—

$$s = \mu \,.\, V^{2\cdot1} \,.\, \left(\frac{1}{R}\right)^{\frac{3}{2}}$$

$$\text{or } V = c\ R^{\frac{5}{7}} \,.\, s^{\frac{10}{21}} \qquad (28)$$

$$\text{where } c = \left(\frac{1}{\mu}\right)^{\frac{10}{21}}$$

If this expression is used instead of $V = c \,.\, R^{\frac{1}{2}} \,.\, S^{\frac{1}{2}}$, the Mississippi gaugings will no longer present anything that is abnormal or discordant with the evidences obtained from smaller channels. For the two or three examples quoted in Table 3 of Art. 29, it appeared that log. μ should be estimated at $\bar{4}{\cdot}270$ to $\bar{4}{\cdot}350$, or $e = 60$ to 55; but it is reasonable to expect that a river-bed will often present obstructions much greater, and relatively greater, than the pebbles in Bazin's artificial channel; so that μ would be considerably greater, and e considerably less than these values, if the river-bed were very irregular.

Hence, if we proceed to deduce the experimental value of this coefficient e from a number of river-gaugings, we certainly cannot expect to find it a constant quantity in all river-beds; but it will not rise to extravagant heights like the coefficient c, whenever the slope is very flat or the radius very large.

Throughout all the examples which have been grouped together in Table 26 it varies only between 50 and 60, while the old coefficient ranges from 66 to upwards of 200. The formula which seems to apply to a mill-race 1 foot in depth applies equally well to a depth of 74 feet; while the slopes range from 1 in 500 to about 1 in 200,000, and the results included in the table do not indicate that any correction is needed either for changes of depth or changes of slope.

But the additional examples given in the next Table, 27, and in Table 28 indicate just as clearly that in other rivers, or in other stretches of the same river, the coefficient e will have lower values. In each group, however, it does not appear that the value is governed by changes of depth or of slope, and it is not unreasonable to suppose that they depend chiefly upon the varying character of the bed.

It must be remarked, however, that many other river-gaugings have been recorded beyond the number that are quoted in these tables; and in *a few* of them the coefficient e comes out at abnormal values, sometimes higher than 60 and sometimes lower than 30. That such divergencies should be found is not at all surprising when we consider the enormous differences between the rocky bed of a mountain rill, the tortuous course of streams in sandy plains, and the broad, deep channel of a great river like the Mississippi.

In every case it is doubtless true that the water-particle obeys one universal law, but the universality of law counts for nothing

when the ultimate result depends upon such diversified conditions.

For this reason it does not seem advisable to accept any river-gaugings as a guide in dealing with regular channels. If we take pipe experiments as the basis for pipe rules, and channel experiments for the framing of channel rules, we shall be following the surest guides; while rivers can hardly be expected to follow any rules that we can frame.

For the same reason we can only apply the channel rules safely within certain limits which do not go far beyond the limits of experiment. In using a formula which has been advocated by Hagen, Lampe, Prof. Unwin, and Prof. Reynolds, we cannot venture to regard it as anything more than an empirical approximation to the truth—true, or nearly true, within certain limits; but not universally true for channels of unlimited dimensions. The Mississippi gaugings, however, may at least serve to show that our calculations have not *over*-estimated the discharge in large channels, and in this connection they have a special value. Such deep river gaugings are in fact the only experiments that can furnish any kind of check upon the accuracy of our formula when it is applied to very large artificial channels.

If we take for this purpose the Mississippi gauging, No. 336, as a fair average example, the hydraulic depth being 63·4 feet, we may compare the observed velocity of current with that which would be given by the formula for a semicircular channel having the same hydraulic depth and the same gradient. Its diameter would be about 253 feet; and if this imaginary channel were lined with either of the materials named in Classes 2, 4, 8, or 15 of Table 3, the calculated velocity would not be much in excess of the velocity here recorded. With an assumed lining of neat cement it comes out at 6·15; for the smoothest brick at 5·63; for rough brickwork at 5·04; and for rubble masonry at 4·96. As the river itself, with an unknown irregularity of bed, flows at the velocity of 5·08 feet per second, it seems probable that these calculations will not be much above the truth when they are applied to a designed channel of greater magnitude than any that have formed the subject of Bazin's experiments.

TABLE 26.—RIVER-GAUGINGS, GIVING EXPERIMENTAL VALUES OF COEFFICIENT e IN FORMULA $V = e \, . \, R^{\frac{5}{7}} \, . \, s^{\frac{10}{21}}$, AND c IN $V = c \, . \, R^{\frac{1}{2}} s^{\frac{1}{2}}$. CLASS 14A. RIVER-BEDS OF LOW RESISTANCE.

No.	Experiment.	R	s	V	Coefficients e	Coefficients c
321	A mill-race at Kagiswyl ...	1·04	0·0017540	2·817	56·0	65·8
322	Ditto	1·11	0·0010000	2·200	54·7	65·7
323	Gürben Canal	2·38	0·0020000	4·789	50·0	69·5
324	The Ohio at Point Pleasant	6·72	0·0000933	2·515	53·5	100·4
325	The Reuss at Mellingen ...	6·95	0·0001500	3·018	50·0	93·2
326	Bayou La Fourche (Hum-	12·80	0·0000365	2·807	59·0	129·7
327	phreys and Abbot)	13·04	0·0000373	2·843	58·2	128·8
328		12·47	0·0000438	2·789	55·3	119·3
329		15·71	0·0000447	3·076	50·6	116·1
330	Great Nevka at St. Petersburg	17·42	0·0000149	2·049	52·9	127·3
331	The Neva	35·42	0·0000139	3·230	52·0	145·6
332	Mississippi at Carrolton ...	57·20	0·0000139	4·460	50·8	158·2
333	(River Commission Re-	59·30	0·0000112	4·050	50·0	157·2
334	port of 1882[1])	62·60	0·0000165	5·240	51·7	163·0
335		63·10	0·0000165	5·900	57·8	182·9
336		63·40	0·0000127	5·080	56·0	179·0
337		65·60	0·0000142	5·130	52·5	168·1
338	Ditto (Humphreys & Abbot)	72·46	0·0000171	5·887	51·3	167·1
339		73·53	0·0000034	4·034		254·4
	or possible slope		0·0000054		60·0	202·5
340		74·39	0·0000038	3·978		235·3
	or possible slope		0·0000058		56·7	191·5

[1] The gaugings at Carrolton include nine others in which the depth R varies from 57 to 64 feet, and the coefficient e from 42 to 50.

TABLE 27.—RIVER-GAUGINGS, GIVING EXPERIMENTAL VALUES OF COEFFICIENT e IN FORMULA $V = e \, . \, R^{\frac{5}{7}} \, . \, s^{\frac{10}{21}}$, AND OF c IN $V = c \, . \, R^{\frac{1}{2}} s^{\frac{1}{2}}$. CLASS 14B. RIVER-BEDS OF MEAN RESISTANCE.

No.	Experiment.	R	s	V	Coefficients e	Coefficients c
341	Saalach in Bavaria ...	1·310	0·0011000	2·240	47·3	58·8
342		1·540	0·0008750	2·073	43·7	56·5
343		1·910	0·0012420	3·077	46·9	63·0
344		1·980	0·0012400	3·385	50·1	68·2
345		2·160	0·0036000	5·474	46·0	64·3

TABLE 27.—*continued.*

No.	Experiment.	R	s	V	Coefficients e	Coefficients c
346	Lauter Canal at Neuburg	1·820	0·0006640	2·106	44·8	61·0
347	Escher Canal	3·760	0·0030000	6·986	43·2	65·7
348		4·420	0·0030000	8·364	46·0	72·6
349	Rhine at Flurlingen ...	6·732	0·0001573	2·965	49·1	91·8
350	Rhine at Noll	7·000	0·0001618	2·834	48·8	84·2
351	Seine at Paris, between the	5·660	0·0001270	2·093	43·5	78·1
352	bridges Jena and Inva-	7·080	0·0001330	2·264	39·2	73·7
353	lids	8·430	0·0001350	2·418	36·8	71·7
354		9·480	0·0001400	3·370	46·2	92·5
355		10·920	0·0001400	3·740	46·4	95·6
356		12·190	0·0001400	3·816	43·8	92·4
357		14·500	0·0001400	4·232	42·8	94·0
358		15·020	0·0001400	4·511	44·5	98·3
359		15·930	0·0001720	4·682	40·2	89·5
360		16·850	0·0001310	4·800	45·0	102·1
361		18·390	0·0001030	4·689	46·4	107·6
362	Seine at Meulan	7·100	0·0000900	2·310	48·1	91·3
363		7·680	0·0000870	2·313	46·3	89·5
364	Seine at Triel	11·240	0·0000570	2·362	44·1	93·3
365		12·430	0·0000600	2·359	40·0	86·4
366	Seine at Poissy	13·570	0·0000500	2·372	43·5	91·1
367		14·200	0·0000540	2·595	42·0	93·6
368		15·860	0·0000620	2·910	40·5	92·7
369		16·850	0·0000670	3·101	40·0	92·4
370	Irawadi at Saiktha (Gor-	17·520	0·0000129	1·459	40·2	97·0
371	don, 1873[1])	19·880	0·0000215	2·083	41·1	100·7
372		20·400	0·0000301	2·620	43·2	105·7
373		22·970	0·0000387	3·091	41·6	103·6
374		26·420	0·0000473	3·548	39·3	100·3
375		29·800	0·0000560	3·993	37·4	97·8
376		33·570	0·0000646	4·432	35·6	95·2
377		37·310	0·0000732	4·874	34·3	93·3
378		41·010	0·0000818	5·382	33·5	92·9
379		44·470	0·0000904	6·147	34·5	97·0
380	Mississippi above Vicks-	31·160	0·0000223	3·523	49·5	133·8
381	burg (Humphreys and	52·120	0·0000303	5·558	46·9	139·9
382	Abbot, 1858)	57·370	0·0000481	6·319	40·0	120·3
383		64·100	0·0000638	6·950	36·0	108·7
384		64·520	0·0000436	6·825	41·6	128·6

[1] The gaugings recorded by Mr. Gordon are twice as numerous. Every second observation is given in this table.

TABLE 28.—RIVER-GAUGINGS, GIVING EXPERIMENTAL VALUES OF COEFFICIENT e IN FORMULA $V = e \, . \, R^{\frac{5}{7}} \, . \, s^{\frac{10}{21}}$ AND OF c IN $V = c \, . \, R^{\frac{1}{2}} \, . \, s^{\frac{1}{2}}$. CLASS 14C. RIVER-BEDS OF HIGH RESISTANCE.

No.	Experiment.	R	s	V	Coefficients e	Coefficients c
385	Rhine in Domleschger Valley	0·25	0·0057750	1·250	39·0	32·8
386		1·32	0·0077350	4·753	39·5	47·0
387		2·95	0·0079590	7·419	34·3	48·3
388	Emme near Emmermatt ...	1·19	0·0050000	3·510	37·0	45·4
389	Lütschine near Eybridge ...	1·34	0·0033250	3·214	39·5	48·0
390	Saare near Laupen Bridge...	2·70	0·0033330	4·559	34·0	48·1
391	Tessin at Giubasco ...	2·96	0·0002540	1·663	39·5	60·6
392	Limmat near Zurich ...	3·16	0·0027500	5·346	39·0	57·4
393	Scheuss Canal near Biel ...	4·35	0·0018500	5·445	38·2	60·8
394	Aar near Thalgut	4·58	0·0017760	5·445	37·5	61·0
395		7·06	0·0017760	6·770	34·3	60·5
396	Danube at Ravensburg ...	5·77	0·0005360	3·762	38·8	67·3
397	Danube at Sarengrad ...	14·25	0·0000580	2·493	38·9	86·9
398	Rhine at Neuburg	13·91	0·0003910	5·838	37·4	78·9
399	Rhine at Pfortz	13·94	0·0003570	5·642	37·7	79·8
400	Mississippi at Fulton (River	29·60	0·0000144	2·200	39·5	106·2
401	Commission Report, 1881[1])	30·40	0·0000187	2·350	36·6	98·6
402		32·90	0·0000209	2·820	39·3	107·4
403		39·30	0·0000495	4·040	33·4	91·6
404		40·10	0·0000476	4·220	34·6	96·6
405		41·10	0·0000513	4·490	34·8	97·8
406	Mississippi at Columbus (Humphreys and Abbot, 1858)	65·88	0·0000680	6·958	34·0	103·9

The examples which have been brought together in these tables may suffice to show what are the *most usual* values of the coefficient e in the given formula. It is, however, not possible to make any general use of them for calculating river discharges.

[1] The Report includes two other gaugings, in which the depth was from 53 to 54 feet, and the coefficient e from 42 to 44.

CHAPTER IX.

CONCLUSIONS.

Art. 68. The Evidence of River-Gaugings.—The only evidence that we possess in regard to the flow of water in large channels is to be found in the deep-water river-gaugings referred to in the previous chapter; and the examples quoted in Tables 26, 27, 28 appear to confirm the general truth of the formula $s = \mu\frac{V^n}{R^m}$, and especially in respect of the relationship between the fall s and the hydraulic depth R.

The admirable experiments of M. Bazin were conducted upon too small a scale to enable him to determine this relationship with precision, although his formula is abundantly accurate for channels of small dimensions. Reverting for a moment to the binomial expression, $s = \frac{V^2}{R}\left(\alpha' + \frac{\beta'}{R}\right)$, quoted in Art. 25, and using the values for earthen channels as given in Table 2, the formula implies that the coefficient c in the old formula[1] could never rise above the value $c = 108{\cdot}5$, however great might be the depth of the channel. This inference is evidently not in agreement with the gaugings quoted in Table 26, which show a continuous increase in the value of c as the depth of the channel increases.

On the other hand, if we make s proportional to $\left(\frac{1}{R}\right)^m$, where m is greater than unity, the statement implies that there is no such limit to the value of the old coefficient. The difference between the two formulæ has been illustrated by the diagram Fig. 15, in which the height OG represents the limiting value of A or $\frac{1}{c^2}$ for channels of infinite depth; and the question will be further illustrated if we compute the value of c by the binomial expression, and compare it with the observed values as given by

[1] The coefficient c is the square root of the reciprocal of M. Bazin's coefficient A, which is calculated by $A = \alpha' + \frac{\beta'}{R}$.

some of the deep-water gaugings in Table 26. Taking experiments Nos. 329 to 340, the comparison stands as follows:—

No.	Experiment.	R	Coefficient c calculated	Coefficient c observed
329	Bayou La Fourche ...	15·71	96·7	116·1
330	Great Nevka	17·42	97·6	127·3
331	The Neva	35·42	102·6	145·6
332	The Mississippi at Carrolton	57·20	104·6	158·2
333	(River Commission)	59·30	104·8	157·2
334		62·60	105·0	163·0
335		63·10	105·0	182·9
336		63·40	105·0	179·0
337		65·60	105·1	168·1
338	Ditto (Humphreys & Abbot)	72·50	105·6	167·1
339		73·50	105·6	202–254
340		74·40	105·6	191–235

At the depth of 15 feet the formula gives a coefficient not much below the observed value; but as the hydraulic depth increases from this point onwards, the calculated coefficient approaches gradually towards the maximum value of 108·5, while the actual gaugings give a series of values rising rapidly above the calculated figures as the depth increases from 15 to 75 feet. The facts, so far as they are exhibited by these examples, are more nearly represented by the curve *OC* in Fig. 15 than by the straight line *GC*; and they are evidently inconsistent with the idea of a limiting maximum value. At the same time, it is quite impossible to deduce any general law from the whole range of river-gaugings.

Art. 69. Completion of the Formula.—With the river-gaugings we come to an end of all the experimental evidence which is at present available for studying the flow of water at uniform speed. In circular pipes and in regular artificial channels of moderate dimensions, it has been found that the relations between the measurable quantities, s, R, and V, are in all cases expressible (so far as they are known) by the general formula, $s = \mu\frac{V^n}{R^m}$; and when the experiment is extended to channels in which R is very much larger than in any pipe or artificial channel, the formula is, at all events, not inconsistent with the observed flow in great rivers.

In every case, however, the constants, μ, n, and m, will depend upon the roughness of the bed. It has been seen that, in pipes and in regular channels, they remain constant for all ordinary values of s, R, and V, so long as the roughness remains constant; while every change in this unmeasurable roughness appears to involve a corresponding change in *each one* of the quantities, μ, n, and m.

In Chap. IV. an endeavour was made to find the true values of these constants in the only way by which they can safely be determined—namely, by direct reference to recorded experiment. By the method described in that chapter it was possible to determine pretty closely the exponent n for Class 1 in Table 3, and approximately also for Class 4; while the exhaustive experiments of M. Bazin enabled us to find each one of the three quantities, for Classes 7, 10, and 13, with a tolerable degree of accuracy. But in dealing with the other materials of construction, classified in Table 3 according to their roughness, it was not possible to carry out the same analysis of experimental results. Thus, for example, in Classes 8, 8A, and 9, we have nowhere the record of any such experiments, in conduits of ordinary brick and ashlar, as would enable us to distinguish between the effects of a variation of slope, and those due to a variation of hydraulic depth.

In default of such experiments, therefore, it seemed best to place conduits of ordinary brick and ashlar in one group with Class 7, whose frictional resistance is nearly the same. Thus for all materials of Group III. we took provisionally $s = \mu \frac{V^{1\cdot8}}{R^{1\cdot2}}$, and the coefficient μ was then found for each one of the materials by reference to actual experiment.

It is perhaps hardly possible, with the experimental knowledge that we now possess, to go farther than this with any confidence; but if we look at the series of figures which have been deduced, in Table 3, from actual experiment, it seems highly probable that the three quantities μ, n, and m, are connected by a constant relationship.

Taking first the ascertained values of n and of log. μ in each of the Classes 7, 10, and 13, and plotting them as co-ordinates, it will be seen that the diagram thus produced is very nearly a straight line, so that the relation between those two quantities is fairly well expressed by writing—

$$\log. \mu = \log. a'' + n \log. \beta''$$

in which a'' and β'' are constants; and so far as these experiments go, it appears that log. a'' is pretty nearly $\bar{6}\cdot00$, or $a'' = \frac{1}{1000000}$, while log. β'' is nearly $1\cdot100$, or $\beta'' = 12\cdot59$, so that we might almost venture to write for channels of rectangular or polygonal section—

$$\log. \mu = \bar{6}\cdot00 + 1\cdot10n$$
$$\text{or } n = \frac{\log. \mu + 6\cdot00}{1\cdot10}$$

For conduits of circular or semicircular section we have seen that μ has always a smaller value, and referring for this purpose to the figures deduced for Classes 1, 4, and 6, we find approximately—

$$\log. \mu = \bar{6}\cdot00 + 1\cdot06n$$
$$\text{or } n = \frac{\log. \mu + 6\cdot00}{1\cdot06}$$

In view of this relationship as shown broadly by the different groups, it is probable that the same relation exists between the separate materials whose surfaces exhibit intermediate degrees of roughness. It is probable, for example, that for the conduits of ordinary brick and ashlar in Classes 8, 8A, and 9, the true value of the exponent n may not be exactly 1·80 as determined for Class 7, but will rather lie between 1·80 and 1·90. In the same way we might almost venture to interpolate values of n between 1·90 and 2·10 for the separate materials of Classes 11, 11A, and 12.

Turning next to the exponent m, we have already seen that, through all the groups of Table 3, it differs but little from $\frac{2}{3}n$.

The ratio $\frac{m}{n}$ or $\frac{1}{\phi}$ was found to have this value in all materials which present a fairly regular surface, including every class from 1 to 9; and it is these gaugings which give the most consistent and reliable results.

Adopting this ratio throughout, our expression would take the more general form—

$$\log. s = \log. a'' + x\left\{\log. \beta'' + \log. \mathrm{V} - \frac{1}{\phi}\log. \mathrm{R}\right\} \qquad (29)$$

$$\text{or } s = a'' \ \beta^x \ \mathrm{V}^x . \left(\frac{1}{\mathrm{R}}\right)^{\frac{x}{\phi}} . \qquad . \quad . \quad . \quad (30)$$

For circular pipes or channels of curved section this would become approximately—

$$\log. s = \bar{6}\cdot00 + x\,\{1\cdot06 + \log. \mathrm{V} - \tfrac{2}{3}\log. \mathrm{R}\} \quad . \quad . \quad (30a)$$

and for channels of rectangular section—

$$\log. s = \bar{6}\cdot00 + x\{1\cdot10 + \log. V - \tfrac{2}{3}\log. R\}. \quad (30b)$$

In these expressions, the variable quantities μ and m have disappeared, while x takes the place of the exponent n in our previous formula, and becomes the only quantity that depends upon the roughness of the bed. This exponent x must, of course, have a separate value for each kind of material, rising by a continuous gradation as the roughness increases, and not advancing in sudden jumps from one group to the next.

The actual value of x for each kind of material must certainly be determined by experiment; but if the formula is in proper agreement with the facts, the exponent x must remain constant through any series of experiments conducted in channels of the same *material*, whatever may be the form, size, or inclination of the conduit. To calculate the exponent from recorded pipe-gaugings, we have for circular sections—

$$x = \frac{\log. s + 6\cdot00}{\log. V + 1\cdot06 - \tfrac{2}{3}\log. R} \quad (31)$$

or in rectangular channels—

$$x = \frac{\log. s + 6\cdot00}{\log. V + 1\cdot10 - \tfrac{2}{3}\log. R} \quad . \ (31a)$$

and the correctness of the general formula can be tested by applying these calculations to each and all of the 250 experiments recorded in Tables 1 to 11.

For each series of gaugings, taken in any one conduit, it will be found that x is very nearly constant, the variations being seldom greater than $\pm$ 0·02, and generally less.[1] The mean value of n for each *series* of gaugings is given in Table 29, and shows very little variation as between conduits of circular and rectangular form, or as between any series of gaugings taken at different slopes.

Taking in view the whole range of these experimental facts, it may be said that they are represented a little more closely by formula (30) with these deduced values of x, than they are by the previous formula with the separate values of μ, n, and m, as found for each group in Table 3. In all ordinary cases, the difference between the two formulæ is really insignificant; but formula (30)

[1] Such a variation in x would evidently entail a difference of about 4 per cent. in calculating the gradient or the total fall, or a difference of about 2½ per cent. in calculating the velocity or the discharge.

possesses the theoretical advantage that in *all* cases (including the most extreme conditions) it keeps always the same order between the relative hydraulic resistances of any two materials, of which one is a little rougher than the other.

But for the practical purposes of the engineer, notwithstanding this advantage, it may be doubted whether it is really worth while to employ an exponent which runs to three or four places of decimals; and for this reason we may justly prefer to take the round numbers given in Table 3, as we have done in working out the several examples and problems in Hydraulic Calculations.

TABLE 29.—CALCULATED VALUES OF x IN THE FORMULÆ

Log. $s = \bar{6}\cdot00 + x(1\cdot06 + \log. V - \frac{2}{3}\log. R)$ for circular sections.
Log. $s = \bar{6}\cdot00 + x(1\cdot10 + \log. V - \frac{2}{3}\log. R)$ for rectangular ditto.

Nos.	Experiment.	x
1–5	Circular, small glass tube, $\frac{7}{8}''$ diam.	1·759
6–9	„ wrought iron, glazed lining, $1\frac{1}{8}''$ diam.	1·762
10–19	„ „ „ $3\frac{1}{4}''$ „	1·753
20–23	„ Dantzic water main „ $16\frac{1}{2}''$ „	1·777
24–27	„ Rosemary pipe „ $48''$ „	1·756
	Mean for pipes of Class 1...	1·761
28–39	Semicircle, channel of neat cement, 49″ diam.	1·7350
40–51	Rectangle, „ „ 6′ 0″ wide	1·7360
	Mean for lining of neat cement (2 and 2A)	1·7355
52–63	Semicircle, channel of cement and sand, 49″ diam.	1·773
64–72	Circular, Sudbury culvert, smooth brick, 9′ diam.	1·768
73–80	„ „ „ „	1·764
81–87	„ „ „ „	1·766
	Mean for smoothest hard brick (4) ...	1·766
88	Rectangle, Roquefavour aqueduct, smooth brick with cemented floor	1·791
89	Rectangle, Aqueduct de Cran, very smooth ashlar masonry	1·745
	Mean value	1·768

TABLE 29.—*continued.*

Nos.	Experiment.	x
90–94	Circular, riveted wrought-iron pipe	1·768
95	„ „ „ Texas, 17″ diam.	1·806
96–104	„ cast-iron bare pipe, 19″ diam. ...	1·800
	Mean for pipes of bare metal (Class 6) ...	1·791
105	Circular, stoneware socket pipe, 18″ diam. ...	1·790
106–117	Rectangle, channel of unplaned timber ...	1·809
118–129	„ „ „ ...	1·805
130–141	„ „ „ ...	1·803
	Mean for unplaned timber (Class 7) ...	1·806
142–153	Rectangle, channel of ordinary brickwork (8A)	1·833
154–157	Rectangle, Chazilly Canal, rough ashlar (9) ...	1·847
158–164	Rectangle, timber with close spaced lathes (10)	1·880
165–171	„ „ „ „	1·909
172–178	„ „ „ „	1·920
	Mean value	1·903
179–188	Semicircle, channel lined with small pebbles (11)	1·940
189–200	Rectangle, „ „ (11A)	1·973
	Mean value, pebbles $\frac{3}{8}''$ to $\frac{7}{8}''$ diam. ...	1·956
201–204	Rectangle, channel in hammer-dressed masonry (12)	1·9780
205–208	„ „ „	1·9770
	Mean value	1·9775
209–215	Rectangle, timber with open-spaced lathes (13)	2·098
216–223	„ „ „	2·102
224–230	„ „ „	2·108
	Mean value	2·103
231–242	Rectangle, channel lined with coarse pebbles (14)	2·083
243–244	Rectangle, channel of dry rubble masonry (15)	2·134
245–247	Semicircle, „ „	2·227
248–250	Rectangle, „ „	2·158
	Mean value for dry rubble	2·173

INDEX AND LIST OF TABLES

END OF PART II.

PRINTED BY WILLIAM CLOWES AND SONS, LIMITED, LONDON AND BECCLES.